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Review

Heat transfer characteristics of nanofluids: a review

Xiang-Qi Wang, Arun S. Mujumdar *

Department of Mechanical Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore, 119260

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Abstract

Research in convective heat transfer using suspensions of nanometer-sized solid particles in base liquids started only over the past decade. Recent investigations on nanofluids, as such suspensions are often called, indicate that the suspended nanoparticles markedly change the transport properties and heat transfer characteristics of the suspension. This review summarizes recent research on fluid flow and heat transfer characteristics of nanofluids in forced and free convection flows and identifies opportunities for future research.

Keywords: Nanofluids; Nanoparticles; Heat transfer; Thermal conductivity

Contents

1. Introduction

Convective heat transfer can be enhanced passively by changing flow geometry, boundary conditions, or by enhancing thermal conductivity of the fluid. Various techniques have been proposed to enhance the heat transfer performance of fluids. Researchers have also tried to increase the thermal conductivity of base fluids by suspending micro- or larger-sized solid particles in fluids since the thermal conductivity of solid is typically higher than that of liquids, seen from Table 1. Numerous theoretical and experimental studies of suspensions containing solid particles have been conducted since Maxwell's theoretical work was published more than 100 years ago [1]. However, due to the large size and high density of the particles, there is no good way to prevent the solid particles from settling out of suspension. The lack of stability of such suspensions, induces additional flow resistance and possible erosion. Hence, fluids

* Corresponding author. Tel.: +65 6874 4623; fax: +65 6779 1459.
E-mail address: mpeasm@nus.edu.sg (A.S. Mujumdar).

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Nomenclature

A	constant
$B_{2,x}$	depolarization factor along x – symmetrical axis
D	diameter of tube ..... mm
d	diameter of particle
H	height of discs ..... mm
h	heat transfer coefficient ..... $W/m^2 K$
k	thermal conductivity ..... W/m K
k_B	Boltzmann constant, $k_B = 1.381 \times 10^{-23}$ J/K
$k_{c,j}$	effective dielectric constant
L	length of tube ..... mm
m	constant
n	empirical shape factor, $n = 3/\psi$
$n(r)$	radius distribution function
Nu	Nusselt number, $Nu = hD/k$
Pe	Peclet number, $Pe = \rho\mu L/\Gamma$
Pr	Prandtl number, $Pr = C_p\mu/k$
r_c	apparent radius of clusters
Re	Reynolds number, $Re = \rho V D/\mu$
T	temperature ..... K
x	coordinate

Greek symbols

α	aspect ratio of nanoparticles
β	ratio of the nanolayer thickness to the original particle radius, $\beta = h/r$
γ	ratio of nanolayer thermal conductivity to particle thermal conductivity, $\gamma = k_{layer}/k_p$
ν	dynamic viscosity
ϕ	volume fraction of nanoparticles in suspension
ψ	particle sphericity

Subscripts

b	base fluid
cl	cluster of particles
eff	effective
in	inner tube
nf	nanofluid
out	outer tube
pe	modified nanoparticle
p	nanoparticle

with dispersed coarse-grained particles have not yet been commercialized.

Modern nanotechnology provides new opportunities to process and produce materials with average crystallite sizes below 50 nm. Fluids with nanoparticles suspended in them are called nanofluids, a term proposed by Choi in 1995 of the Argonne National Laboratory, U.S.A. [2]. Nanofluids can be considered to be the next-generation heat transfer fluids as they offer exciting new possibilities to enhance heat transfer performance compared to pure liquids. They are expected to have superior properties compared to conventional heat transfer fluids, as well as fluids containing micro-sized metallic particles. The much larger relative surface area of nanoparticles, compared to those of conventional particles, should not only significantly improve heat transfer capabilities, but also should increase the stability of the suspensions. Also, nanofluids can improve abrasion-related properties as compared to the conventional solid/fluid mixtures. Successful employment of nanofluids will support the current trend toward component miniaturization by enabling the design of smaller and lighter heat exchanger systems. Keblinski et al. [3] made an interesting simple review to discuss the properties of nanofluids and future challenges. The development of nanofluids is still hindered by several factors such as the lack of agreement between results, poor characterization of suspensions, and the lack of theoretical understanding of the mechanisms.

Suspended nanoparticles in various base fluids can alter the fluid flow and heat transfer characteristics of the base fluids. Necessary studies need to be carried out before wide application can be found for nanofluids. In this paper we present an overview of the literature dealing with recent developments in the study of heat transfer using nanofluids. First, the preparation of nanofluids is discussed; this is followed with a review of recent experimental and analytical investigations with nanofluids.

Table 1
Thermal conductivities of various solids and liquids

	Material	Thermal conductivity (W/m K)
Metallic solids	copper	401
	aluminum	237
Nonmetallic solids	silicon	148
	alumina (Al_2O_3)	40
Metallic liquids	sodium (644 K)	72.3
Nonmetallic liquids	water	0.613
	ethylene glycol (EG)	0.253
	engine oil (EO)	0.145

2. Preparation of nanofluids

Preparation of nanofluids is the first key step in experimental studies with nanofluids. Nanofluids are not simply liquid-solid mixtures. Some special requirements are essential e.g. even and stable suspension, durable suspension, negligible agglomeration of particles, no chemical change of the fluid, etc. Nanofluids are produced by dispersing nanometer-scale solid particles into base liquids such as water, ethylene glycol (EG), oils, etc. In the synthesis of nanofluids, agglomeration is a major problem. There are mainly two techniques used to produce nanofluids: the single-step and the two-step method. The single-step direct evaporation approach was developed by Akoh et al. [4] and is called the VEROS (Vacuum Evaporation onto a Running Oil Substrate) technique. The original idea of this method was to produce nanoparticles, but it is difficult to subsequently separate the particles from the fluids to produce dry nanoparticles.

A modified VEROS process was proposed by Wagener et al. [5]. They employed high pressure magnetron sputtering for the preparation of suspensions with metal nanoparticles such as Ag and Fe. Eastman et al. [6] developed a modified VEROS technique, in which Cu vapor is directly condensed into nanoparticles by contact with a flowing low-vapor-pressure liquid (EG).

Zhu et al. [7] presented a novel one-step chemical method for preparing copper nanofluids by reducing $CuSO_4 \cdot 5H_2O$ with $NaH_2PO_2 \cdot H_2O$ in ethylene glycol under microwave irradiation. Results showed that the addition of $NaH_2PO_2 \cdot H_2O$ and the adoption of microwave irradiation are two significant factors which affect the reaction rate and the properties of Cu nanofluids.

A vacuum-SANSS (submerged arc nanoparticle synthesis system) method has been employed by Lo et al. [8] to prepare Cu-based nanofluids with different dielectric liquids such as de-ionized water, with 30%, 50%, 70% volume solutions of ethylene glycol and pure ethylene glycol. They found that the different morphologies, which are obtained, are mainly influenced and determined by the thermal conductivity of the dielectric liquids. CuO, Cu_2O, and Cu based nanofluids also can be prepared by this technique efficiently. An advantage of the one-step technique is that nanoparticle agglomeration is minimized, while the disadvantage is that only low vapor pressure fluids are compatible with such a process. Recently, a Ni nanomagnetic fluid was also produced by Lo et al. [9] using the SANSS method.

The two-step method is extensively used in the synthesis of nanofluids considering the available commercial nanopowders supplied by several companies. In this method, nanoparticles was first produced and then dispersed the base fluids. Generally, ultrasonic equipment is used to intensively disperse the particles and reduce the agglomeration of particles. For example, Eastman et al. [6], Lee et al. [10], and Wang et al. [11] used this method to produce Al_2O_3 nanofluids. Also, Murshed et al. [12] prepared TiO_2 suspension in water using the two-step method. Other nanoparticles reported in the literature are gold (Au), silver (Ag), silica and carbon nanotubes. As compared to the single-step method, the two-step technique works well for oxide nanoparticles, while it is less successful with metallic particles.

Except for the use of ultrasonic equipment, some other techniques such as control of pH or addition of surface active agents, are also used to attain stability of the suspension of the nanofluids against sedimentation. These methods change the surface properties of the suspended particles and thus suppress the tendency to form particle clusters. It should be noted that the selection of surfactants should depend mainly on the properties of the solutions and particles. Xuan and Li [13] chose salt and oleic acid as the dispersant to enhance the stability of transformer oil–Cu and water–Cu nanofluids, respectively. Oleic acid and cetyltrimethylammoniumbromide (CTAB) surfactants were used by Murshed et al. [12] to ensure better stability and proper dispersion of TiO_2–water nanofluids. Sodium dodecyl sulfate (SDS) was used by Hwang et al. [14] during the preparation of water-based MWCNT nanofluids since the fibers are entangled in the aqueous suspension.

In general, methods such as change of pH value, addition of dispersant, and ultrasonic vibration aim at changing the surface properties of suspended particles and suppressing formation of particles cluster to obtain stable suspensions. However, the addition of dispersants can affect the heat transfer performance of the nanofluids, especially at high temperature.

3. Experimental Investigations

3.1. Measurement of thermal conductivity

Since thermal conductivity is the most important parameter responsible for enhanced heat transfer many experimental works been reported on this aspect. The transient hot wire method [15], the steady-state parallel-plate technique [11] and the temperature oscillation technique [16] have been employed to measure the thermal conductivity of nanofluids. Among them the transient hot wire method has been used most extensively. Because in general nanofluids are electrically conductive, it is difficult to apply the ordinary transient hot-wire technique directly. A modified hot-wire cell and electrical system was proposed by Nagasaka and Nagashima [17] by coating the hot wire with an epoxy adhesive which has excellent electrical insulation and heat conduction. However, Das et al. [16] pointed that possible concentration of ions of the conducting fluids around the hot wire may affect the accuracy of such experimental results.

The oscillation method was proposed by Roetzel et al. [18] and further developed by Czarnetski and Roetzel [19]. This method is purely thermal and the electrical components of the apparatus are removed from the test sample. Hence ion movement should not affect the measurement.

Alumina (Al_2O_3) and copper oxide are the most common and inexpensive nanoparticles used by many researchers in their experimental investigations. All the experimental results have demonstrated the enhancement of the thermal conductivity by addition of nanoparticles. Eastman et al. [6] measured the thermal conductivity of nanofluids containing Al_2O_3, CuO, and Cu nanoparticles with two different base fluids: water and HE-200 oil. A 60% improvement of the thermal conductivity was achieved as compared to the corresponding base fluids for only 5 vol% of nanoparticles. They also showed that the use of Cu nanoparticles (using the one-step method) results in larger improvements than that of CuO (using the two-step method).

Lee et al. [10] suspended CuO and Al_2O_3 (18.6 and 23,6 nm, 24.4 and 38.4 nm for them, respectively) with two different base fluids: water and ethylene glycol (EG) and obtained four combinations of nanofluids: CuO in water, CuO in EG, Al_2O_3 in water and Al_2O_3 in EG. Their experimental results showed that nanofluids have substantially higher thermal conductivities than the same liquids without nanoparticles. The CuO/EG mixture showed enhancement of more than 20% at 4 vol% of nanoparticles. In the low volume fraction range (<0.05 in test), the thermal conductivity ratios increase almost linearly with volume fraction. Although the size of Al_2O_3 particle is smaller than that of CuO, CuO-nanofluids exhibited better thermal conductivity values than Al_2O_3-nanofluids; no explanation is available for this observation at this time.

Wang et al. [11] measured the effective thermal conductivity of nanofluids by a steady-state parallel-plate technique. The base fluids (water, ethylene glycol (EG), vacuum pump oil and engine oil) contained suspended Al_2O_3 and CuO nanoparticles of 28 and 23 nm of average diameters, respectively. Experimental results demonstrated that the thermal conductivities of all nanofluids were higher than those of their base fluids. Also, comparison with various data indicated that the thermal conductivity of nanofluids increases with decreasing particles size. Results demonstrated 12% improvement of the effective thermal conductivity at 3 vol% of nanoparticles as compared to 20% improvement reported by Masuda et al. [20] and 8% reported by Lee et al. [10] at the same volume fraction of particles.

Xuan and Li [13] enhanced the thermal conductivity of water using Cu particles of comparatively large size (100 nm) to the same extent as has been found using CuO particles of much smaller dimension (36 nm). A appropriate selection dispersants may improve the stability of the suspension. They used oleic acid for transformer oil–Cu nanofluids and laurate salt for water–Cu suspension in their study and found that Cu particles in transformer oil had superior characteristics to the suspension of Cu particles in water.

Xie et al. [21] investigated the effects of the pH value of the suspension, the specific surface area (SSA) of the dispersed Al_2O_3 particles, the crystalline phase of the solid phase, and the thermal conductivity of the base fluid on the thermal conductivity of nanofluids. They found that the increase in the difference between the pH value and isoelectric point (the pH at which a molecule carries no net electrical charge) of Al_2O_3 resulted in enhancement of the effective thermal conductivity. Also, the thermal conductivity enhancements were highly dependent on the specific surface area (SSA) of the nanoparticles. The crystalline phase of the nanoparticles did not appear to have any obvious effect on the thermal conductivity of the suspensions.

Eastman et al. [22] used pure Cu nanoparticles of less than 10 nm size and achieved 40% increase in thermal conductivity for only 0.3% volume fraction of the solid dispersed in ethylene glycol. They indicated that the increased ratio of surface to volume with decreasing size should be an important factor. Also, they showed that the additive acid may stabilize the suspension and thus increase the effective thermal conductivity.

A Fe-nanofluid was prepared by Hong and Yang [23] with ethylene glycol; Fe nanoparticles with mean size of 10 nm were produced by chemical vapor condensation process. They found that Fe nanofluids exhibited higher enhancement of thermal conductivity than Cu nanofluids. Their result indicated that the material with high thermal conductivity is not always the best candidate for the suspension to improve the thermal characteristics of base fluids. Also, they concluded that the thermal conductivity of nanofluids increased non-linearly with the solid volume fraction. Hong et al. [24] also investigated the effect of the clustering of Fe nanoparticles on the thermal conductivity of nanofluids. They found that the thermal conductivity of nanofluids is directly related to the agglomeration of Fe nanoparticles, which caused the nonlinear relation between the Fe volume fraction and thermal conductivity of nanofluids due to rapid clustering of nanoparticles in condensed nanofluids.

Murshed et al. [12] investigated TiO_2 nanoparticles in rod-shape ($\varnothing 10 \times 40$) and spherical shape ($\varnothing 15$) dispersed in deionized water. They observed that nearly 33% and 30% enhancement of the effective thermal conductivity occurred for TiO_2 particles of $\varnothing 10 \times 40$ and $\varnothing 15$, respectively. Results showed that both particle size and shape influence the thermal conductivity of nanofluids.

Xie et al. [25,26] prepared and measured the thermal conductivities of 26 nm and 0.6 μm SiC suspensions in deionized water and EG using a transient hot-wire method. Different from experimental results of Lee at al. [10], they found that the nanofluids with the same solid particles in different base fluids had the same improvement in the effective thermal conductivity. Furthermore, results showed that HC model [27] is capable of predicting the thermal conductivity of 0.6 μm SiC suspensions, while it under-predict that of 26 nm particles.

Das et al. [16] examined the effect of temperature on thermal conductivity enhancement for nanofluids containing Al_2O_3 (38.4 nm) or CuO (28.6 nm) through an experimental investigation using temperature oscillation method. They observed that a 2 to 4-fold increase in thermal conductivity can take place over the temperature range of 21 °C to 52 °C. The results suggest the application of nanofluids as cooling fluids for devices with high energy density where the cooling fluid is likely to work at a temperature higher than the room temperature. They also mention that the inherently stochastic motion of nanoparticles could be a probable explanation for the thermal conductivity enhancement since smaller particles show greater enhancements of thermal conductivity with temperature than do larger particles.

Li and Peterson [28] conducted an experimental investigation to examine the effects of variations in the temperature and volume fraction on the effective thermal conductivity of CuO (29 nm) and Al_2O_3 (36 nm) water suspensions. Results demonstrated that nanoparticle material, diameter, volume fraction and bulk temperature have significant effects on the thermal conductivity of the nanofluids. For example, for Al_2O_3/water suspension, increase in the mean temperature from 27 to 34.7 °C results in the enhancement of nearly three times. They also derived two simple two-factor linear regression for the discussed nanofluids (Al_2O_3/water: $(k_{\text{eff}} - k_{\text{b}})/k_{\text{b}} = 0.764\phi + 0.0187(T - 273.15) - 0.462$, CuO/water: $(k_{\text{eff}} - k_{\text{b}})/k_{\text{b}} = 3.761\phi + 0.0179(T - 273.15) - 0.307$). However, additional investigations are necessary to verify the impact of the temperature on the effective thermal conductivity of nanofluids.

Patel et al. [29] studied gold (Au) and silver (Ag) nanoparticles with thoriate and citrate as coatings in water- and toluene-based fluids. The nanofluids were prepared to check the conductivity enhancement effect at low concentrations They found 5%–21% enhancement of the thermal conductivity of nanofluids for water with citrate in the temperature range 30–60 °C at a very low loading of 0.00026 vol% of Ag particles. For a loading of 0.011% of Au particles, the improvement of thermal conductivity was around 7%–14%. Such interesting phenomena indicate that, except for particle size, there exist important

factor related to the motion of particles. Also, the increments in thermal conductivity of the nanofluids were found to be nonlinear with temperature and almost linear with particle volume fraction. The important chemical factors such as the need for direct contact of the metal surface with the solvent medium, have important effects on the resulting effective thermal conductivity.

Recently, however, Putnam et al. [30] did not observe significant enhancement in the thermal conductivity of nanofluids with small volume fractions of nanoparticles such as C_{60}–C_{70} and Au ($\phi \ll 1$). The observed largest increase in thermal conductivity for 4 nm Au particles is $1.3\% \pm 0.8\%$, which was conflicted with the anomalous results of Patel et al. [29]. Further quantitative measurements need to be carried out to explain reasons for this contradiction.

The largest increases in thermal conductivity have been observed in suspensions of carbon nanotubes, which have very high aspect ratio and very high thermal conductivity. The first report on the synthesis of nanotubes was conducted by Iijima [31]. Later, nanotube (multiwalled carbon nanotubes or MWNTs)-oil (α-olefin) mixtures were investigated by Choi et al. [32] to measure their effective thermal conductivity. Results displayed that the measured thermal conductivity was anomalously greater than the theoretical predictions and was nonlinear with nanotube loadings. As compared to other nanostructured materials discussed previously, the nanotubes achieved the highest conductivity enhancement and provided wide opportunities for effective management applications. Xie et al. [33] also proposed a method to produce stable and homogeneous suspensions of multiwalled carbon nanotubes (CNTs) in deionized water (DW), ethylene glycol (EG), and decene (DE). They introduced oxygen-containing functional groups on CNT surfaces to form more hydrophilic surfaces. Experimental data indicated that the thermal conductivity enhancement increased with increase in nanotube loading, but decreased with thermal conductivity increase of the base fluid.

Biercuk et al. [34] measured the effective thermal conductivity of suspensions of single wall carbon nanotubes (SWNTs) and vapor grown carbon fibers (VGCF) in epoxy using a comparative method [35]. Results showed 125% and 45% improvements for 1.0 wt% SWNTs and VGCF, respectively. They found Choi et al. [36] on thermal properties of SWNTs-epoxy composites showed similar improvement of the thermal conductivity. They pointed out that the bundling of nanotubes could be an important factor for thermal transport characteristics.

Wen and Ding [37] investigated the effect of temperature on the thermal conductivity of MCNTs (20–60 nm in diameter and a few tens of micrometers in length)/water nanofluids. For temperatures lower than 30 °C, an approximately linear dependence of thermal conductivity enhancement on temperature was obtained. However, the dependence levels off when temperature was higher than 30 °C. Ding et al. [38] also showed that the effective thermal conductivity increases with increasing temperature in CNT–water suspensions. They found that the improvement of the thermal conductivity is slightly higher than that reported by Assael et al. [39], Xie et al. [33], and Wen and Ding [37], but much lower than that showed in Choi et al. [32]. The discrepancy among the different groups should rely on the properties of CNTs used, the aspect ratio, the inclusion of dispersants, and the experimental errors involved.

Assael et al. [39,40] experimentally studied the enhancement of the thermal conductivity of carbon-multiwall nanotubes (C-MWNT)–water suspensions with 0.1 wt% sodium dodecyl sulfate (SDS) as a dispersant. They found that the maximum thermal conductivity enhancement was 38% for a 0.6 vol% suspension. Results showed that the additional SDS would interact with C-MWNT in that the outer surface was affected. Later, Assael et al. [41] repeated the similar measurements using carbon-multiwall nanotubes (C-MWNTs) and carbon double-walled nanotubes (C-DWNTs), but using hexadecyltrimethyl ammonium bromide (CTAB) and nanosphere AQ as dispersants instead. The maximum thermal conductivity enhancement obtained was 34% for a 0.6 vol% C-MWNT–water suspension with CTAB. They also discussed the effect of surfactant concentration on the effective thermal conductivity of the suspensions and found that CTAB is better for C-MWNTs and C-DWNTs.

Recently, Liu et al. [42] measured the thermal conductivities of nanofluids containing CNTs dispersed in ethylene glycol and a synthetic engine oil. The increase of thermal conductivity is up to 12.4% for CNT–ethylene glycol suspensions at 1.0 vol% and 30% for CNT–synthetic engine oil suspensions at 2 vol%. The results from Liu [42] are relatively lower than other data as shown in Fig. 2. It can be seen that the data from different groups vary widely and it is difficult to get a regression directly from the available data for CNT. One possible reason for this is that the thermal conductivity is highly dependent on such important factors such as the structure of the CNTs, clustering, temperature, etc. Further systematic research is necessary to obtain a whole map for the thermal conductivities of CNTs.

Hwang et al. [14] compared the thermal conductivity of four kinds of nanofluids such as MWCNTs in water, CuO in water, SiO_2 in water, and CuO in ethylene glycol. They found that the thermal conductivity of MWCNT nanofluid was increased up to 11.3% at 1 vol%, which is relatively higher than that of the other groups of nanofluids.

From the aforementioned discussion, which is also summarized in Table 2, we find that the available experimental data from different research groups vary widely, as shown in Figs. 1 and 2. Further investigations are necessary to clarify the current predicament.

3.2. Measurement of viscosity

Compared with the experimental studies on thermal conductivity of nanofluids, there are limited rheological studies reported in the literature. Li et al. [43] measured the viscosity of water with CuO nanoparticle suspensions using a capillary viscometer. Results showed that the apparent viscosity of nanofluids decreased with increasing temperature. However, as they pointed out, the capillary tube diameter may influence the apparent viscosity for higher nanoparticle mass fractions, especially at lower temperatures.

Wang et al. [11] also measured the relative viscosity of Al_2O_3–water and Al_2O_3–ethylene glycol nanofluids. Results

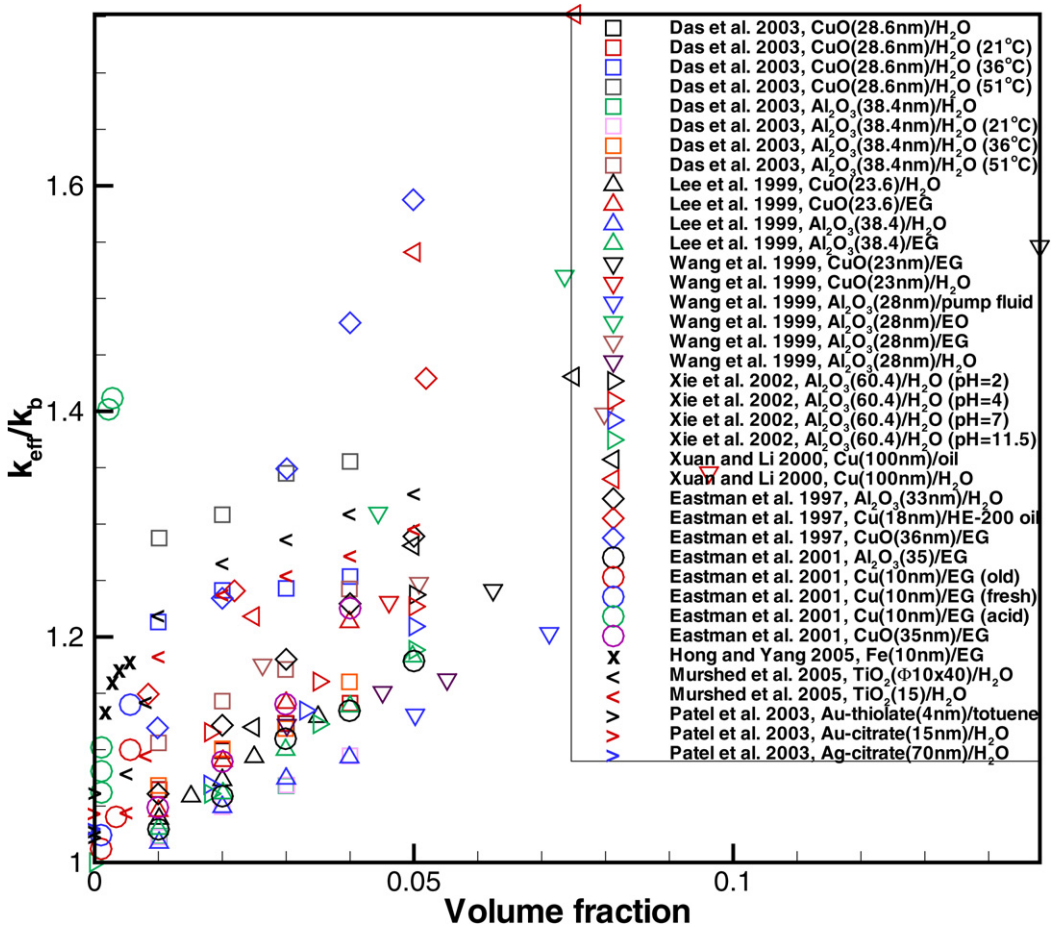

Fig. 1. Comparison of experimental data on thermal conductivity of nanofluids.

showed similar trend of increase of relative viscosity with increased solid volume fraction for the two nanofluids. That means the desirable heat transfer increase may be offset by the undesirable increase in pressure drop.

Das et al. [44] also measured the viscosity of Al_2O_3–water nanofluids against shear rate. Their results showed an increase of viscosity with increased particle concentrations. There is strong possibility that nanofluid may be non-Newtonian; even viscoelastic in some cases. Further experimental studies are needed to define the viscosity models of nanofluids so they can be used in simulation studies.

The viscosity of CNT–water nanofluids as a function of shear rate was measured by Ding et al. [38] recently. They observed that the viscosity of nanofluids increased with increasing CNT concentration and decreasing temperature. Also, the shear thinning behavior was found by the authors. That means the nanofluids can provide better fluid flow performance due to the higher shear rate at the wall, which results in low viscosity there.

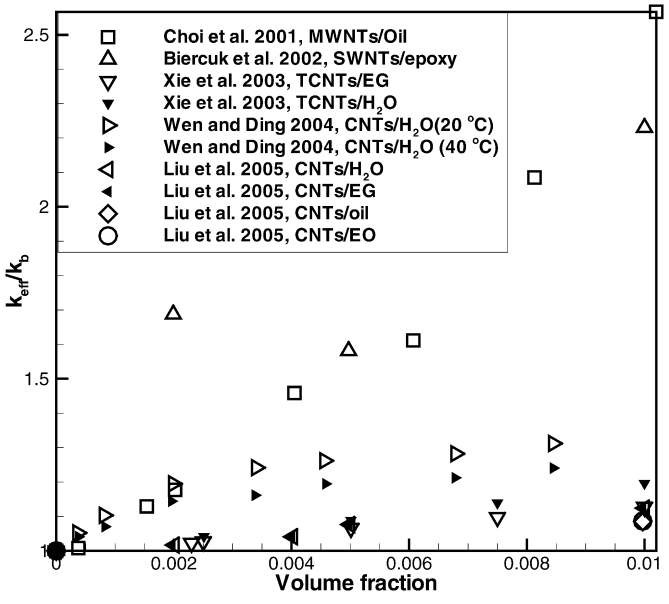

Fig. 2. Comparison of some experimental data on thermal conductivity for carbon nanotube-based nanofluids.

3.3. Convective heat transfer

Past decade has seen many research activities in heat transfer characteristics of various nanofluids experimentally. For forced convective heat transfer, Lee and Choi [45] studied the heat transfer behavior in parallel channels using an unspecified nanofluid and observed a reduction in thermal resistance by a factor of 2. Xuan and Li [46] experimentally investigated flow

Table 2
Summary of experimental studies on thermal conductivity of nanofluids

Investigator	Particles	Size (nm)	Fluids	Observations
Eastman et al. [6]	Al_2O_3/CuO/Cu	33/36/18	water, HE-200 oil	60% improvement for 5 vol% CuO particles in water
Lee et al. [10]	Al_2O_3/CuO	24.4,38.4/18.6,23.6	water, EG	20% improvement for 4 vol% Cuo/EG mixture
Wang et al. [11]	Al_2O_3/CuO	28/23	water, EG, PO, EO	12% improvement for 3 vol% Al_2O_3/water nanofluids
Das et al. [16]	Al_2O_3/CuO	38.4/28.6	water	2–4 fold increase over range of 21 °C to 52 °C
Xie et al. [21]	Al_2O_3	12.2–302	water, EG, PO	pH value, SSA, crystalline phase
Li and Peterson [28]	Al_2O_3/CuO	36/29	water	enhancement with volume fraction and temperature
Xuan and Li [13]	Cu	100	water, oil	successful suspension of relatively big metallic nanoparticles
Eastman et al. [22]	Cu	<10	EG	40% increase for 0.3 vol% Cu-based nanofluids
Hong and Yang [23]	Fe	10	EG	18% increase for 0.55 vol% Fe/EG nanofluids
Patel et al. [29]	Au, Ag	4, 15/70	water, toluene	size, temperature, and chemical characteristics
Murshed et al. [12]	TiO_2	∅10 × 40, ∅15	DW	33% and 30% increase at 5 vol% for ∅10 × 40 and ∅15, respectively
Xie et al. [21,25]	SiC	∅26, 600	water, EG	15.8% increase at 4.2 vol% for ∅26 SiC–H_2O and 22.9% at 4 vol% for ∅600 SiC–H_2O
Choi et al. [32]	MWNTs	∅25 × 50 μm	oil	exceed 250% at 1.0 vol%
Biercuk et al. [34]	SWNTs	∅3–30	epoxy	125% at 1.0 wt%
Xie et al. [33]	TCNTs	∅15 × 30 μm	DW, EG, DE	19.6%, 12.7%, and 7.0% increase at 1.0 vol% for TCNT/DE, EG, and DW, respectively
Choi et al. [36]	SWNTs	∅20–30 × 200	epoxy	300% at 3 wt% SWNT loading
Wen and Ding [37]	CNTs	∅20–60× ∼ 10 μm	water	23.7% ans 31% increase at 0.84% CNT concentration for 20 °C and 45 °C, respectively
Assael et al. [39–41]	MWNTs, DWNTs	∅130 × 10 μm	water	34% increase for 0.6 vol% suspension
Liu et al. [42]	CNTs	∅20–30	EG, EO	12.4% for EG at 1 vol%, 30% for EO at 2 vol%

Note: EG: ethylene glycol; PO: pump oil; EO: engine oil;DW: deionized water; DE: decene.

and convective heat transfer characteristics for Cu–water based nanofluids through a straight tube with a constant heat flux at wall. Results showed that the nanofluids give substantial enhancement of heat transfer rate compared to pure water. They also claimed that the friction factor for the nanofluids at low volume fraction did not produce extra penalty in the pumping power.

Wen and Ding [47] reported experimental results for the convective heat transfer of γ-Al_2O_3 (27–56 nm)/water based nanofluids flowing through a copper tube ($D = 4.5$ mm, $L = 970$ mm) in laminar regime. They found that the inclusion of Al_2O_3 particles can significantly enhance the convective heat transfer coefficient, which increases with increasing Reynolds number and particle concentrations. Furthermore, the improvement of the heat transfer coefficient was particularly large in the entrance region, and decreased with the axial distance. Apart from the improved effective thermal conductivity, they also attributed the improvement of heat transfer to particle migration, which caused a non-uniform distribution of thermal conductivity and viscosity field along the cross-section in the tube.

Heris et al. [48] investigated laminar flow of CuO/water and Al_2O_3/water nanofluids through a 1 m annular copper tube with 6 mm inner diameter and with 0.5 mm thickness and 32 mm diameter outer stainless steel tube, where saturated steam was circulated to create constant wall temperature boundary condition rather than constant heat flux condition by other researchers. Comparison of experimental results showed that the heat transfer coefficient enhanced with increasing volume fraction of nanoparticles as well as Peclet number while Al_2O_3/water showed more enhancement.

Chien et al. [49] investigated gold (17 nm)/water nanofluids flowing in a disk-shaped miniature heat pipe with diameter of 9 mm and height of 2 mm. Their data showed that the thermal resistance of heat pipe fell appreciably with increased nanoparticle concentration. Tsai et al. [50] also employed aqueous solutions of various-sized (2–35 nm and 15–75 nm) gold nanoparticles, which were prepared by the reduction of $HAuCl_4$ with trisodium citrate and tannic acid. They found a large decrease of thermal resistance of the heat pipe with nanofluids as compared with de-ionized water. The thermal resistance of the circular heat pipe ranged from 0.17 to 0.215 K/W with different nanoparticle solutions. The reason is that the included nanoparticles can bombard the vapor during the bubble formation. Hence, the reduction of thermal resistance was obtained due to the resulted smaller bubble size. Results indicated the high potential of nanofluids as working medium to replace the conventional fluids in heat pipes. Ma et al. [51] combined nanofluids with oscillating heat pipe (OHP) to develop an ultrahigh-performance cooling device. Experimental results showed that the diamond nanofluid could reduce the temperature difference between the evaporator and the condenser from 40.9 to 24.3 °C for 80 W input power.

Ding et al. [38] investigated the heat transfer performance of CNT nanofluids in a tube with 4.5 mm inner diameter. They found that the observed enhancement of heat transfer coefficient is much higher than the increase in the effective thermal conductivity. They associated the possible reasons with the improved thermal conductivity, shear-induced enhancement in flow, reduced boundary layer, particle re-arrangement, and high aspect ratio of CNTs. These observations suggest that the aspect ratio should be associated with the high enhancement of heat transfer performance of CNTs-based nanofluids.

However, there are some inconsistent reports on nanofluid behavior in forced convection. Pak and Cho [52] studied heat transfer performance of γ-Al_2O_3 – (13 nm) and TiO_2 – (27 nm) water based nanofluids in tube. They found that the convective

heat transfer coefficient of the nanofluids at $\phi = 3$ vol% was 12% lower than that of pure water for a constant average velocity. The possible reason is that the suspensions have higher viscosity than that of pure water, especially at high particle volume fractions.

Similar trends were observed by Yang et al. [53], who investigated the convection heat transfer characteristics of the graphite nanofluids in laminar flow through a circular tube with diameter of 4.57 mm and length of 457 mm. Note that the particles they used are disc-like (the average diameter is 1–2 μm with the thickness around 20–40 nm). Unexpectedly, the experimental results showed that the increase of the heat transfer coefficient of the system is much lower than the enhancement of the effective thermal conductivity itself. That means apart from the effective thermal conductivities, particle shape or aspect ratio (0.02 here) of the nanoparticle should be an important factor in determining the thermal performance of nanofluids, which also can be seen in the CNT-based suspensions with very high aspect ratio [38] (>100). Further investigation should be necessary to clarify this problem.

Previous studies with nearly spherical nanoparticles (aspect ratio, α 1) [46,47,52] showed an enhancement of the convective heat transfer of up to 60%. Results on CNTs nanofluids ($\alpha \gg 1$) [38] increased the convective heat transfer coefficient over 350% at $Re = 800$ for 0.5 wt% CNTs. However, the disc-shape nanoparticle (α 0.02) from Yang et al. showed much lower increase of convective heat transfer coefficient with respect to the effective thermal conductivity. What we can conclude from the available experimental data is that the particle shape or aspect ratio of the particle is a significant parameter to affect the thermal performance of nanofluids. However, it has not been well examined yet.

For natural convective heat transfer, relatively few investigations have been carried out. As discussed previously in Section 3.3, Khanafer et al. [54] numerically investigated the heat transfer behavior of nanofluids in a two-dimensional horizontal enclosure. The nanofluids were assumed to be in single phase, in thermal equilibrium and without velocity slip between base fluid and particle. It was showed that the heat transfer rate increased with the particle concentration at any given Grashof number. However, different experimental results have been observed by Putra et al. [55] and Wen and Ding [56].

Putra et al. [55] presented their experimental observations on natural convection of Al_2O_3 and CuO–water nanofluids inside a horizontal cylinder heated from one end and cooled from the other. Unlike the results of forced convection, they found a systematic and definite deterioration of the natural convective heat transfer, which was dependent on the particle density, concentration, and the aspect ratio of the cylinder. The deterioration increased with particle concentration and was more significant for CuO nanofluids. For example, at Rayleigh number of 5×10^7, 300% and 150% decrease in the Nusselt number was found for 4 wt% of CuO and Al_2O_3, respectively. Then, what caused the different results between the numerical results [54] and the experimental data [55]? It should be clarified that in the numerical study presented by Khanafer et al. [54], some important factors were not included. The factors include the particle size, particle shape, and particle distribution, which could significantly influence the flow and heat transfer characteristics of nanofluids. However, these factors has not been investigated properly as far.

Recently, Wen and Ding [56] also addressed the problem of natural convective heat transfer of TiO_2 (30–40 nm)/water nanofluids in a vessel which was composed of two horizontal aluminum discs of diameter 240 mm and thickness 10 mm separated by a 10 mm gap. They investigated both the transient and steady heat transfer coefficients for various concentrations of nanofluids. Similar to Putra et al. [55], they also found that the natural convective heat transfer coefficient decreased as compared to that of pure water. Furthermore, such deterioration increased with nanoparticle concentrations. They proposed several possible mechanisms for their observations such as the convection caused by concentration difference, particle–fluid and particle–particle interactions, and modifications of the dispersion properties.

Considering the limited experimental studies on the natural convection heat transfer in nanofluids as listed in Table 3, firm conclusions can not be drawn yet. However, it has been clearly shown by the available results that the heat transfer behavior of nanofluids is very complex and the application of nanofluids for heat transfer enhancement should not be decided only by their effective thermal conductivity. Many other factors such as particle size, shape and distribution, micro-convection, pH value, and the particle–fluid interactions should have important influence on the heat transfer performance of the nanofluids in natural convective heat transfer, which should be identified further in future work.

3.4. Boiling heat transfer

Continuous advances in semiconductor miniaturization and manufacturing are bringing power densities to increasingly higher levels. For example, at the upper limit of future applications, high-end military and aerospace band-gap amplifier will produce waste heat flux on the order of 1000 W/cm^2. Only two-phase (boiling) liquids are suitable for such high dissipation rates. Faulkner et al. [57] tried to achieve 1000 W/cm^2 cooling flux using boiling with ceramic/water nanofluids. Their maximum heat flux dissipation was only 125 W/cm^2 for saturated boiling and 280 W/cm^2 for sub-cooled boiling. From their results we can see the high potential of boiling of nanofluids in cooling systems.

Witharana [58] investigated the boiling heat transfer coefficients (HTC) of Au (unspecified size)/water, SiO_2 (30 nm)/water, and SiO_2/ethylene glycol nanofluids in a cylindrical vessel with 10 cm in diameter and 10 cm in height. The bottom of the vessel was supplied by fixed heat flux and the top was open to the atmosphere. Results of Au/water nanofluids ($\phi = 0.0002$ 0.001 wt%) showed that the HTC of nanofluids was higher than that of pure water, and increased with increasing gold particle concentrations. For example, the enhancement of HTC was above 11% in the intermediate heat flux (3 W/cm^2) and as high as 21% in the extreme case (4 W/cm^2). However, the SiO_2/water and SiO_2/ethylene glycol nanofluids recorded

Table 3
Summary of experiments on convective heat transfer of nanofluids

Investigator	Geometry	Nanofluids	Findings
Forced convective heat transfer:			
Lee and Choi [45]	parallel channels	unspecified	reduction in thermal resistance by a factor of 2
Xuan and Li [46]	tube ($D = 10$, $L = 800$ mm),	Cu/water	turbulent, large enhancement of heat transfer coef. $Nu_{nf} = c_1(1.0 + c_2\phi^{m_1} Pe_d^{m_2})Re_{nf}^{m_3} Pr_{nf}^{0.4}$
Wen and Ding [47]	tube ($D = 4.5$, $L = 970$ mm)	Al_2O_3/water (27–56 nm)	laminar, enhancement increases with Reynolds number and particle concentration
Chien et al. [49]	disk-shaped heat pipe ($D = 9$, $H = 2$ mm)	Au/water (17 nm)	significant reduction of thermal resistance
Tsai et al. [50]	heat pipe ($D = 6$, $L = 170$ mm)	Au/water (2–35, 15–75 nm)	high potential to take place conventional fluids in heat pipe applications
Ding et al. [38]	tube ($D = 4.5$, $L = 970$ mm)	CNT/water	significant enhancement of convective heat transfer, which depends on the flow condition, CNT concentration and the pH level
Pak and Cho [52]	tube	Al_2O_3 (13 nm), TiO_2 (27 nm)/water	h with $\phi = 0.03$ vol% was 12% lower than that of pure water for a given average fluid velocity
Yang et al. [53]	tube ($D = 4.57$, $L = 457$ mm)	graphite nanofluid	the enhancement of h is lower than the increase of the effective thermal conductivity
Heris et al. [48]	annular tube ($D_{in} = 1$ mm, $D_{out} = 32$ mm, $L = 1$ m)	Al_2O_3(20 nm), CuO (50–60 nm)/water	enhancement of h with ϕ and Pe. Al_2O_3 showed more enhancement than CuO
Natural convective heat transfer:			
Putra et al. [55]	horizontal cylinder	CuO (87.3 nm), Al_2O_3 (131.2 nm)/water	a systematic and significant deterioration in natural convective heat transfer
Wen and Ding [56]	two horizontal discs ($H = 10$, $D = 240$ mm)	TiO_2/water (30–40 nm)	deterioration increases with particle concentrations

decreased HTC as compared to the base fluids, which was somewhat contrary to expectations. The author had not explained such strange phenomena. Possibly re-examination of the experiments should be a good choice. Li et al. [59] also observed deteriorated pool boiling heat transfer for CuO/water nanofluids. They attributed the reason to the decreasing of active nucleation sites from nanoparticle sedimentation.

Das et al. [44] carried out an experimental study of pool boiling characteristics of Al_2O_3 nanofluids under atmospheric conditions on a tube with diameter in 20 mm. The found that the inclusion of nanoparticles degraded the boiling performance by increasing the wall superheat for a given heat flux. The deterioration in boiling performance increased with increasing particle concentration and surface roughness. This means there should be additional effects to degrade the boiling characteristics such as the changed surface features of the nanoparticles. Since the surface tension and latent heat were unaffected and the only unfavorable change was the increased viscosity, the heat transfer characteristics during pool boiling was expected to be enhanced considering the significant increase of thermal conductivity, which had active effects on the major factors in heat transfer during pool boiling such as the micro-layer evaporation and reformation of thermal boundary layer. They attributed it to the affected surface roughness during pool boiling of nanofluids. For higher particle concentration and higher surface roughness, the uneven surface can trap the particles more easily and make the surface smoother, which can cause the degradation of the boiling performance.

Das et al. [60] also studied the pool boiling performance under tubes with small diameter (4, 6.5 mm) where the bubble size and tube diameter are in the same order. They observed that deterioration for the narrow tubes was lower than that in the large tube ($D = 20$ mm). The small tube results in a large curvature of the surface to induce direct departure rather than sliding of larger bubbles. From the discussion above, apart from the increased effective thermal conductivity, there should be some other factors that affect the boiling performance of nanofluids.

Bang and Chang [61,62] studied boiling of Al_2O_3–water nanofluids on a 100 mm square surface at high heat fluxes and observed that the surface roughness after boiling increased with nanoparticle concentration. However, the critical heat flux (CHF) (the peak heat flux, under which a boiling surface can stay in nucleate boiling regime) performance has been enhanced to ∼32% and ∼13% for both horizontal flat surface and vertical flat surface in the pool, respectively. They claimed that the increased roughness caused by the deposition of nanoparticles will cause a fouling effect to deteriorate the boiling heat transfer performance.

On the other hand, however, significant pool boiling heat transfer enhancement was found for Al_2O_3/water nanofluids. by Tu et al. [63]. You et al. [64] investigated the boiling curve and the CHF of Al_2O_3/water nanofluids in pool boiling with various nanoparticle concentrations ranging from 0 g/l to 0.05 g/l. They found that the boiling heat transfer coefficients of all various concentrations as well as pure water were the same, which demonstrated that the nucleate boiling heat transfer efficiency was not affected by the inclusion of nanoparticles. They also found that the size of bubbles increased with addition of nanoparticles to water. Correspondingly, the frequency of bubble departure degraded substantially. They claimed that there were some unknown key factors to increase the CHF in nanofluids, which need further investigation.

Vassallo et al. [65] confirmed that the CHF increases for nanofluids (silica–water). They conducted experiments for both nano- and micro-solutions at the same solid volume fraction on a 0.4 mm diameter horizontal NiCr wire at the atmospheric pressure. The heat transfer enhancement were not found in the nucleate boiling regime, but the CHF was increased significantly for both nano- and micro-particles. Addition of nanoparticles resulted in a maximum heat flux of about three times that of pure water and almost twice that of micro-particle/water mixture.

Zhou [66] investigated experimentally the heat transfer characteristics of copper/acetone based nanofluids with and without acoustic cavitation. Results showed that the copper nanoparticles and acoustic cavitation had significant influence on heat transfer in the fluid. However, the addition of nanoparticles did not affect the dependence of the heat transfer on acoustic cavitation and fluid sub-cooling. As compared to the experimental results of Das et al. [44,60], the pool boiling heat transfer did not reduce with increased particle volume fractions in absence of acoustic field. While in an acoustic field, the boiling heat transfer of nanofluids was enhanced and the boiling hysteresis disappeared.

Wen and Ding [67] conducted experiments on pool boiling heat transfer using γ-Al_2O_3–water nanofluids, which were produced through an electrostatic stabilization method with the aid of a high shear homogeniser. They found that presence of alumina in the nanofluid can enhance the boiling heat transfer significantly, by ∼40% for a 1.25 wt% concentration of the particles. Considering the controversies from previous studies, they proposed some possible reasons such as the extra thermal resistance to the boiling surface caused by the sedimentation of nanoparticles, effect of surfactant, and interaction between boiling surface and nanofluids. The aggregation of nanofluids should be an important factor to affect the boiling performance, which need to be clarified quantitatively further.

The currently available experimental data on boiling heat transfer of nanofluids (as shown in Table 4) are limited. However, conflicting results were observed from these limited data as far as the effect of nanoparticles on the boiling heat transfer performance is concerned. The inconsistencies indicate that our understanding of the thermal behavior of nanofluids related to the boiling heat transfer is still poor. Further detailed and valuable investigations are necessary for us to understand the phenomena of boiling of nanofluids. As we know, the pool boiling will be affected by the surface properties such as surface roughness, surface wettability, and surface contamination. In the reviewed studies, however, only the surface roughness is the most often considered parameter. They systematic studies should have been carried out to include the interaction between the surface and nanofluids (wettability), also as suggested by Wen and Ding [67].

4. Theoretical investigations

4.1. Mechanisms of nanofluids

The conventional understanding of the effective thermal conductivity of mixtures originates from continuum formulations which typically involve only the particle size/shape and volume fraction and assume diffusive heat transfer in both fluid and solid phases. This method can give a good prediction for micrometer or lager-size solid/fluid systems, but it fails to explain the unusual heat transfer characteristics of nanofluids.

To explain the reasons for the anomalous increase of the thermal conductivity in nanofluids, Keblinski et al. [68] and Eastman et al. [69] proposed four possible mechanisms e.g. Brownian motion of the nanoparticles, molecular-level layering of the liquid at the liquid/particle interface, the nature of heat transport in the nanoparticles, and the effects of nanoparticle clustering, which are schematically shown in Fig. 3. They postulated that the effect of Brownian motion can be ignored since

Table 4
Experiments on boiling heat transfer of nanofluids

Investigator	Geometry	Nanofluids	Results
Witharana [58]	cylindrical vessel ($\phi 10 \times 10$ cm)	Au, SiO_2 (30 nm)/water, EG	Au/water nanofluids displayed increased h, but SiO_2/water, SiO_2/EG caused decreased h
Li et al. [59]	–	CuO/water	Deteriorated pool boiling heat transfer
Das et al. [44]	tube ($D = 20$ mm)	Al_2O_3/water	Nanoparticles deteriorated boiling performance and the degradation was found to increase with particle concentrations
Das et al. [60]	tube ($D = 4$, 6.5 mm)	Al_2O_3/water	the deterioration in narrow tube is less
Tu el al. [63]	–	Al_2O_3/water	significant pool boiling enhancement
You et al. [64]	cartridge	Al_2O_3/water	enhancement of CHF 200% for pool boiling, no change for nucleate boiling
Vassallo et al. [65]	NiCr wire ($D = 0.4$ m)	SiO_2/water	markable increase of CHF for both nano- and micro-solutions, but no significant differences for powers less than CHF
Zhou [66]	horizontal copper tube	Cu/acetone (80–100 nm)	Effects of acoustical parameters, nanofluids concentration, and fluid subcooling on heat transfer enhancement
Wen and Ding [67]	cylindrical boiling vessel ($D = 160$ mm, $H = 300$ mm)	Al_2O_3/water	significant enhancement of the boiling heat transfer, ∼40% at 1.25 wt% suspensions
Bang and Chang [61,62]	horizontal heater in a vessel	Al_2O_3/water	decrease of pool nucleate boiling heat transfer, but enhancement of CHF performance

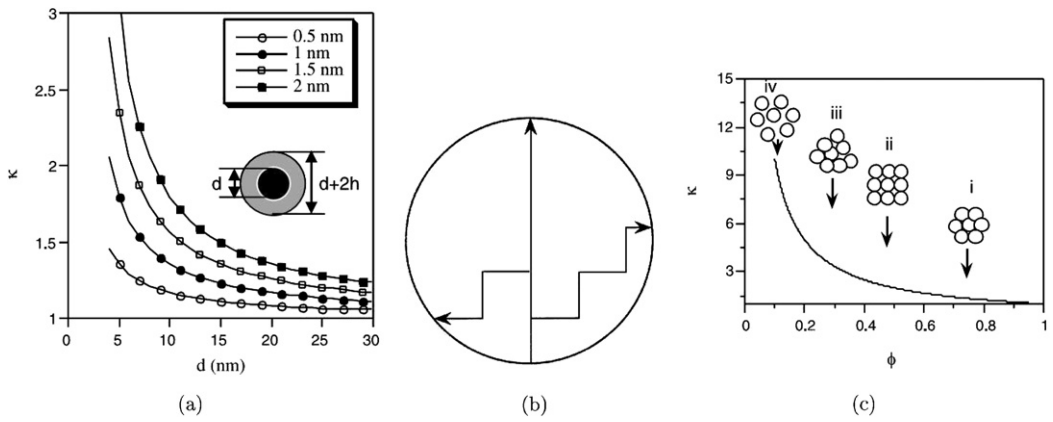

Fig. 3. Schematic diagrams of several possible mechanisms [68]: (a) Enhancement of k due to formation of highly conductive layer-liquid structure at liquid/particle interface; (b) Ballistic and diffusive phonon transport in a solid particle; (c) Enhancement of k due to increased effective ϕ of highly conducting clusters.

contribution of thermal diffusion is much greater than Brownian diffusion. However, they only examined the cases of stationary nanofluids. Wang et al. [11] argued that the thermal conductivities of nanofluids should be dependent on the microscopic motion (Brownian motion and inter-particle forces) and particle structure. Xuan and Li [13] also discussed four possible reasons for the improved effective thermal conductivity of nanofluids: the increased surface area due to suspended nanoparticles, the increased thermal conductivity of the fluid, the interaction and collision among particles, the intensified mixing fluctuation and turbulence of the fluid, and the dispersion of nanoparticles.

Many researchers used the concept of liquid/solid interfacial layer to explain the anomalous improvement of the thermal conductivity in nanofluids. Yu and Choi [70,71] suggested models based on conventional theory which consider a liquid molecular layer around the nanoparticles. However, a study of Xue et al. [72] using molecular dynamics simulation showed that simple monatomic liquids had no effect on the heat transfer characteristics both normal and parallel to the surface. This means that thermal transport in layered liquid may not be adequate to explain the increased thermal conductivity of suspensions of nanoparticles.

Khaled and Vafai [73] investigated the effect of thermal dispersion on heat transfer enhancement of nanofluids. These results showed that the presence of the dispersive elements in the core region did not affect the heat transfer rate. However, the corresponding dispersive elements resulted in 21% improvement of Nusselt number for a uniform tube supplied by a fixed heat flux as compared to the uniform distribution for the dispersive elements. These results provide a possible explanation for the increased thermal conductivity of nanofluids which may determined partially by the dispersive properties.

Wen and Ding [74,75] studied the effect of particle migration on heat transfer characteristics in nanofluids flowing through mini-channels ($D = 1$ mm) theoretically. They studied the effect of shear-induced and viscosity-gradient-induced particle migration and the self-diffusion due to Brownian motion. Their results indicated a significant non-uniformity in particle concentration and thermal conductivity over the tube cross-section due to particle migration. As compared to the uniform distribution of thermal conductivity, the non-uniform distribution caused by particle migration induced a higher Nusselt number.

Koo and Kleinstreuer [76] discussed the effects of Brownian, thermo-phoretic, and osmo-phoretic motions on the effective thermal conductivities. They found that the role of Brownian motion is much more important than the thermo-phoretic and osmo-phoretic motions. Furthermore, the particle interaction can be neglected when the nanofluid concentration is low (<0.5%). However, these findings have not been validated by experiment yet.

Recently, Evans et al. [77] suggested that the contribution of Brownian motion to the thermal conductivity of the nanofluid is very small and cannot be responsible for the extraordinary thermal transport properties of nanofluids. They also supported their argument using the molecular dynamics simulations and the effective medium theory. However, they just limited their discussion to stationary fluids, which weakens their results.

Rather than Brownian motion, liquid layering, phonon transport, and agglomeration, Lee et al. [78] experimentally investigated the effect of surface charge state of the nanoparticle in suspension on the thermal conductivity. They showed that the pH value of the nanofluid strongly affected the thermal performance of the fluid. With farther diverged pH value from the isoelectric point of the particles, the nanoparticles in the suspension got more stable so to change the thermal conductivity. That may partially explain the disparities between different experimental data since many researchers used surfactants in nanofluids, but with insufficient descriptions. By adopting a variation of the classical heat conduction method in porous media to the problem of heat conduction in nanofluids, On the other hand, Vadasz [79] demonstrated that the transient heat conduction process in nanofluids may provide a valid explanation for the apparent heat transfer enhancement.

Hence, so far there are no general mechanisms to rule the strange behavior of nanofluids including the highly improved effective thermal conductivity, although many possible factors have been considered, including Brownian motion, liquid–solid interface layer, ballistic phonon transport, and surface charge state. However, there are still some other possible macro-scale explanations such as heat conduction, particle-driven natural

convection, convection induced by electrophoresis, thermophoresis, etc.

4.2. Thermal conductivity

Currently, there is no reliable theory to predict the anomalous thermal conductivity of nanofluids. From the experimental results of many researchers, it is known that the thermal conductivity of nanofluids depends on parameters including the thermal conductivities of the base fluid and the nanoparticles, the volume fraction, the surface area, and the shape of the nanoparticles, and the temperature. There are no theoretical formulas currently available to predict the thermal conductivity of nanofluids satisfactorily.

However, there exist several semi-empirical correlations to calculate the apparent conductivity of two-phase mixtures. They are mainly based on the following definition of the effective thermal conductivity of a two-component mixture

$$k_{\text{eff}} = \frac{k_{\text{p}}\phi_{\text{p}}(\mathrm{d}T/\mathrm{d}x)_{\text{p}} + k_{\text{b}}\phi_{\text{b}}(\mathrm{d}T/\mathrm{d}x)_{\text{b}}}{\phi_{\text{p}}(\mathrm{d}T/\mathrm{d}x)_{\text{p}} + \phi_{\text{b}}(\mathrm{d}T/\mathrm{d}x)_{\text{b}}} \tag{1}$$

For particle–fluid mixtures, numerous theoretical studies have been conducted dating back to the classical work of Maxwell [1]. The Maxwell model for thermal conductivity for solid–liquid mixtures of relatively large particles (micro-/mini-size) is good for low solid concentrations. The effective thermal conductivity, k_{eff}, is given by

$$k_{\text{eff}} = \frac{k_{\text{p}} + 2k_{\text{b}} + 2(k_{\text{p}} - k_{\text{b}})\phi}{k_{\text{p}} + 2k_{\text{b}} - (k_{\text{p}} - k_{\text{b}})\phi}k_{\text{b}} \tag{2}$$

where k_{p} is the thermal conductivity of the particle, k_{b} is the thermal conductivity of the base fluid and ϕ is the particle volume fraction of the suspension. Maxwell's formula shows that the effective thermal conductivity of nanofluids relies on the thermal conductivity of the spherical particle, the base fluid and the volume fraction of the solid particles.

Bruggeman [80] proposed a model to analyze the interactions among randomly distributed particles. For a binary mixture of homogeneous spherical inclusions, the Bruggeman model gives

$$\phi\left(\frac{k_{\text{p}} - k_{\text{eff}}}{k_{\text{p}} + 2k_{\text{eff}}}\right) + (1-\phi)\left(\frac{k_{\text{b}} - k_{\text{eff}}}{k_{\text{b}} + 2k_{\text{eff}}}\right) = 0 \tag{3}$$

This model can be applied to spherical particles with no limitations on the concentration of inclusions. For low solid concentrations, the Bruggeman model results in almost the same results as the Maxwell model provides. When the particle concentration is sufficiently high, the Maxwell model fails to provide a good match with the experimental results. However, the Bruggeman model agrees quite well with the experimental data [2].

Hamilton and Crosser [27] proposed a model for liquid–solid mixtures for non-spherical particles. They introduced a shape factor, n, to account for the effect of the shape of the particles. The thermal conductivity, in which the ratio of conductivity of the solid and fluid phases is larger than 100 ($k_{\text{p}}/k_{\text{b}} > 100$), can be expressed as follows:

$$k_{\text{eff}} = \frac{k_{\text{p}} + (n-1)k_{\text{b}} - (n-1)(k_{\text{b}} - k_{\text{p}})\phi}{k_{\text{p}} + (n-1)k_{\text{b}} + (k_{\text{b}} - k_{\text{p}})\phi}k_{\text{b}} \tag{4}$$

where n is the empirical shape factor given by $n = 3/\psi$, and ψ is the particle sphericity, defined as the ratio of the surface area of a sphere with volume equal to that of the particle, to the surface area of the particle. Comparison between Eqs. (4) and (2) reveals that Maxwell's model is a special case of the Hamilton and Crosser's model for sphericity equal to one.

As discussed earlier, the classical models are derived from continuum formulations and include only the particle shape and volume fraction as variables and assumed diffusive heat transport in both liquid and solid phases. The large enhancement of the effective thermal conductivity in nanofluids defies Maxwell's theory [1] as well as its modification by Hamilton and Crosser [27]. Some important mechanisms in nanofluids appear to be neglected in these models. Keblinski et al. [68] investigated the possible factors of enhancing thermal conductivity in nanofluids such as the size, the clustering of particles, and the nano-layer between the nanoparticles and base fluids. Based on the traditional models, many later theoretical works have been proposed to address such effects, especially the interfacial characteristics.

Yu and Choi [70] proposed a modified Maxwell model to account for the effect of the nano-layer by replacing the thermal conductivity of solid particles k_{p} in Eq. (2) with the modified thermal conductivity of particles k_{pe}, which is based on the so-called effective medium theory [81]:

$$k_{\text{pe}} = \frac{[2(1+\gamma) + (1+\beta)^3(1+2\gamma)\gamma]}{-(1+\gamma) + (1+\beta)^3(1+2\gamma)}k_{\text{p}} \tag{5}$$

where $\gamma = k_{\text{layer}}/k_{\text{p}}$ is the ratio of nano-layer thermal conductivity to particle thermal conductivity and $\beta = h/r$ is the ratio of the nano-layer thickness to the original particle radius. Hence, the Maxwell equation (Eq. (2)) can be re-cast as follows:

$$k_{\text{eff}} = \frac{k_{\text{pe}} + 2k_{\text{b}} + 2(k_{\text{pe}} - k_{\text{b}})(1+\beta)^3\phi}{k_{\text{pe}} + 2k_{\text{b}} - (k_{\text{pe}} - k_{\text{b}})(1+\beta)^3\phi}k_{\text{b}} \tag{6}$$

The new model including the nano-layer can predict the presence of very thin nano-layers having a thickness less than 10 nm. It also indicates that the addition of smaller (<10 nm) particles could be better than increasing solid volume fraction with respect to the improvement of thermal conductivity.

Yu and Choi [71] proposed a modified Hamilton–Crosser model to include the particle–liquid interfacial layer for non-spherical particles. The effective thermal conductivity was expressed as

$$k_{\text{eff}} = \left(1 + \frac{n\phi_{\text{eff}}A}{1 - \phi_{\text{eff}}A}\right)k_{\text{b}} \tag{7}$$

where A is defined by

$$A = \frac{1}{3}\sum_{j=a,b,c}\frac{(k_{\text{p}j} - k_{\text{b}})}{k_{\text{p}j} + (n-1)k_{\text{b}}}$$

and

$$\phi_{\text{eff}} = \frac{\phi\sqrt{(a^2+t)(b^2+t)(c^2+t)}}{\sqrt{abc}}$$

is the equivalent volume concentration of complex ellipsoids, which is an imaged structure of elliptical particles ($a \geqslant b \geqslant c$) with surrounding nano-layers. With a general empirical shape factor n ($n = 3\Psi^{-\alpha}$, here α is an empirical parameter and Ψ is the particle sphericity), this modified HC model can predict the thermal conductivity of carbon nanotube-in-oil nanofluids reasonably well. However, it fails to predict the nonlinear behavior of the effective thermal conductivity of general oxide and metal based nanofluids.

Xue [82] developed a model for the effective thermal conductivity of nanofluids. His model is based on the average polarization theory and includes the effect of the interface between the solid particles and the base fluid. His formula of effective thermal conductivity is

$$9\left(1-\frac{\phi}{\lambda}\right)\frac{k_{\text{eff}}-k_{\text{b}}}{2k_{\text{eff}}+k_{\text{b}}} + \frac{\phi}{\lambda}\left[\frac{k_{\text{eff}}-k_{\text{c},x}}{k_{\text{eff}}+B_{2,x}(k_{\text{c},x}-k_{\text{eff}})} + 4\frac{k_{\text{eff}}-k_{\text{c},y}}{2k_{\text{eff}}+(1-B_{2,x})(k_{\text{c},y}-k_{\text{eff}})}\right] = 0 \tag{8}$$

where $\lambda = abc/[(a+t)(b+t)(c+t)]$ with half-radii (a, b, c) of the assumed elliptical complex nanoparticles, which consist of nanoparticles and interfacial shells between particles and the base fluids. $k_{\text{c},j}$ is the effective dielectric constant and $B_{2,x}$ is the depolarization factor along x-symmetrical axis which is derived from the average polarization theory. A test of this formula [83] reveals that it is not as accurate as Xue claimed since he used incorrect values of the parameters such as the depolarization factor. Xue and Xu [84] obtained an equation for the effective thermal conductivity according to Bruggeman model [80]. Their model takes account of the effect of interfacial shells by replacing the thermal conductivity of nanoparticles with the assumed thermal conductivity of the so-called "complex nanoparticles", which included the interfacial shells between the nanoparticles and the base fluids.

$$\left(1-\frac{\phi}{\alpha}\right)\frac{k_{\text{eff}}-k_{\text{b}}}{2k_{\text{eff}}+k_{\text{b}}} + \frac{\phi}{\alpha}\frac{(k_{\text{eff}}-k_2)(2k_2+k_1)-\alpha(k_1-k_2)(2k_2+k_{\text{eff}})}{(2k_{\text{eff}}+k_2)(2k_2+k_1)+2\alpha(k_1-k_2)(k_2-k_{\text{eff}})} = 0 \tag{9}$$

where α is the volume ratio of spherical nanoparticle and complex nanoparticle. k_1 and k_2 are the thermal conductivity of the nanoparticle and interfacial shell, respectively. The modified model is in good agreement with the experimental data on the effective thermal conductivity of CuO/water and CuO/EG nanofluids [10].

Xie et al. [85] considered the interfacial nano-layer with linear thermal conductivity distribution and proposed an effective thermal conductivity model to account for the effects of nano-layer thickness, nanoparticles size, volume fraction, and thermal conductivities of fluid, nanoparticles, and nano-layer. Their formula is

$$k_{\text{eff}} = \left(1 + 3\Theta\phi_{\text{T}} + \frac{3\Theta^2\phi_{\text{T}}^2}{1-\Theta\phi_{\text{T}}}\right)k_{\text{b}} \tag{10}$$

with

$$\Theta = \frac{\beta_{\text{lb}}[(1+\gamma)^3 - \beta_{\text{pl}}/\beta_{\text{bl}}]}{(1+\gamma)^3 + 2\beta_{\text{lb}}\beta_{\text{pl}}}$$

where

$$\beta_{\text{lb}} = \frac{k_{\text{l}}-k_{\text{b}}}{k_{\text{l}}+2k_{\text{b}}}$$

$$\beta_{\text{pl}} = \frac{k_{\text{p}}-k_{\text{l}}}{k_{\text{p}}+2k_{\text{l}}}$$

$$\beta_{\text{bl}} = \frac{k_{\text{b}}-k_{\text{l}}}{k_{\text{b}}+2k_{\text{l}}}$$

and $\gamma = \delta/r_{\text{p}}$ is the thickness ratio of nano-layer and nanoparticle. ϕ_{T} is the modified total volume fraction of the original nanoparticle and nano-layer, $\phi_{\text{T}} = \phi(1+\gamma)^3$. They claimed that the calculated values could agree well with some available experimental data.

For metallic particles, Patel et al. [29] found 9% enhancement of thermal conductivity even at extremely low concentrations such as 0.00026%. (Note that if not mentioned in the context, the concentration should be in volume fraction.) The previous formulas fail to predict such strange phenomena. The Brownian motion of nanoparticles at the molecular and nanoscale levels may be a key mechanism governing the thermal behavior of nanofluids. Also, from recent experiments [16] we find that the thermal conductivity of nanofluids depends strongly on temperature. Hence, this important fact should be considered in theoretical models.

Xuan et al. [86] considered the random motion of suspended nanoparticles (Brownian motion) based on the Maxwell model and proposed a modified formula for the effective thermal conductivity as follows:

$$k_{\text{eff}} = \frac{k_{\text{p}}+2k_{\text{b}}-2(k_{\text{b}}-k_{\text{p}})\phi}{k_{\text{p}}+2k_{\text{b}}+(k_{\text{b}}-k_{\text{p}})\phi}k_{\text{b}} + \frac{\rho_{\text{p}}\phi c_{\text{p}}}{2}\sqrt{\frac{k_{\text{B}}T}{3\pi r_{\text{c}}\mu}} \tag{11}$$

where Boltzmann constant $k_{\text{B}} = 1.381 \times 10^{-23}$ J/K, r_{c} is the apparent radius of clusters and depends on the fractal dimension of the cluster structure. Although this model incorporates the effect of temperature on the conductivity enhancement, the dependence is too weak ($\propto T^{1/2}$) and not in agreement with the experimental data of Das et al. [16].

Based on the fractal theory [87], which can well describe the disorder and stochastic process of clustering and polarization of nanoparticles within the mesoscale limit, a fractal model for predicting the effective thermal conductivity of nanofluid was proposed by Wang et al. [88], who developed a fractal model based on the multi-component Maxwell model by substituting the effective thermal conductivity of the particle clusters, $k_{\text{cl}}(r)$, and the radius distribution function, $n(r)$, as follows:

$$k_{\text{eff}} = \frac{(1-\phi)+3\phi\int_0^\infty k_{\text{cl}}(r)n(r)/[k_{\text{cl}}(r)+2k_{\text{b}}]\,\mathrm{d}r}{(1-\phi)+3\phi\int_0^\infty k_{\text{b}}(r)n(r)/[k_{\text{cl}}(r)+2k_{\text{b}}]\,\mathrm{d}r}k_{\text{b}} \tag{12}$$

This model fit successfully the experimental data for 50 nm CuO particle suspension in deionized water with $\phi < 0.5\%$.

Kumar et al. [89] proposed a comprehensive model to account for the large enhancement of thermal conductivity in nanofluids and its strong temperature dependence, which was deduced from the Stokes–Einstein formula. The thermal conductivity enhancement taking account of the Brownian motion of particles can be expressed as:

$$k_{\text{eff}} = k_{\text{b}} + c\frac{2k_{\text{B}}T}{\pi\nu d_{\text{p}}^2}\frac{\phi r_{\text{b}}}{k_{\text{b}}(1-\phi)r_{\text{p}}}k_{\text{b}} \tag{13}$$

where c is a constant, ν is the dynamic viscosity of the base fluid, and d_{p} is the diameter of the particles. However, the validity of the model has got to be established; it may not be suitable for high concentration of particles.

Bhattacharya et al. [90] developed a technique to compute the effective thermal conductivity of a nanofluid using Brownian motion simulation. They combined the liquid conductivity and particle conductivity as follows

$$k_{\text{eff}} = \phi k_{\text{p}} + (1-\phi)k_{\text{b}} \tag{14}$$

where k_{p} is replaced by the effective contribution of the particles towards the overall thermal conductivity of the system, $k_{\text{p}} = \frac{1}{k_{\text{B}}T^2V}\sum_{j=0}^{n}\langle Q(0)Q(j\Delta t)\rangle\Delta t$. The model showed a good agreement of the thermal conductivity of nanofluids.

Jang and Choi [91] devised a theoretical model that involves four modes such as collision between base fluid molecules ($k_{\text{b}}(1-\phi)$), thermal diffusion in nanoparticles in fluids ($k_{\text{p}}\phi$), collision between nanoparticles due to Brownian motion (neglected), and thermal interaction of dynamic or "dancing" nanoparticles with the base fluid molecules ($fh\delta_{\text{T}}$). The resulting expression for the effective thermal conductivity of nanofluids is

$$k_{\text{eff}} = k_{\text{b}}(1-\phi) + k_{\text{p}}\phi + 3C\frac{d_{\text{b}}}{d_{\text{p}}}k_{\text{b}}Re_{d_{\text{p}}}^2 Pr\phi \tag{15}$$

where $h \sim (k_{\text{b}}/d_{\text{p}})Re_{d_{\text{p}}}^2 Pr^2$ and $\delta \sim 3d_{\text{p}}$ presents the heat transfer coefficient for the flow past nanoparticles and the thickness of the thermal boundary layer, respectively. The advantage of the model is to include the effects of concentration, temperature, and particle size. However, the Brownian effect was neglected, which may not be suitable since the high temperature-dependent properties may caused by the Brownian motion.

On the other side, Prasher [30,92] proposed that convection caused by Brownian motion of the nanoparticles is primarily responsible for the enhancement in the effective thermal conductivity of nanofluids. By introducing the general correlation for heat transfer coefficient h, he modified the Maxwell model by including the convection of the liquid near the particles due to Brownian movement:

$$k_{\text{eff}} = \left(1 + A\,Re^m Pr^{0.333}\phi\right)\left[\frac{k_{\text{p}} + 2k_{\text{b}} + 2(k_{\text{p}} - k_{\text{b}})\phi}{k_{\text{p}} + 2k_{\text{b}} - (k_{\text{p}} - k_{\text{b}})\phi}\right]k_{\text{b}} \tag{16}$$

where $h = k_{\text{b}}/a(1 + A\,Re^m\,Pr^{0.333}\phi)$ and A and m are constants. The Reynolds number can be written as:

$$Re = \frac{1}{\nu}\sqrt{\frac{18k_{\text{b}}T}{\pi\rho_{\text{p}}d_{\text{p}}}}$$

Recently, Koo and Kleinstreuer [93,94] developed a new model for nanofluids, which includes the effects of particle size, particle volume fraction and temperature dependence as well as properties of the base fluid and the particle subject to Brownian motion. The resulting formula is

$$k_{\text{eff}} = \frac{k_{\text{p}} + 2k_{\text{b}} + 2(k_{\text{p}} - k_{\text{b}})\phi}{k_{\text{p}} + 2k_{\text{b}} - (k_{\text{p}} - k_{\text{b}})\phi}k_{\text{b}} + 5\times 10^4\beta\phi\rho_{\text{p}}c_{\text{p}}\sqrt{\frac{k_{\text{B}}T}{\rho_{\text{p}}D}}\,f(T,\phi) \tag{17}$$

Note that the first part of Eq. (17) is obtained directly from the Maxwell model while the second part accounts for Brownian motion, which causes the temperature dependence of the effective thermal conductivity. $f(T,\phi)$ can be assumed to vary continuously with the particle volume fraction, $f(T,\phi) = (-6.04\phi + 0.4705)T + (1722.3\phi - 134.63)$ while β is related to particle motion. Based on the investigation of pressure gradients, temperature profiles and Nusselt numbers, Koo and Kleinstreuer [94] also claimed that addition of 1–4% CuO nanoparticles and high-Prandtl number base fluid such as ethylene glycol and oils could significantly increase the heat transfer performance of micro-heat sinks.

Considering the fact that carbon nanotubes possess large aspect ratio, their thermal conductivity is more difficult to predict. Nan et al. [95] generalized Maxwell–Garnett approximation and derived a simple formula ($k_{\text{eff}} = 1 + \phi k_{\text{p}}/3k_{\text{b}}$) to predict the effective thermal conductivity of carbon-nanotube-based composites. The results within Nan's model [95] agree well with the experimental observations [32]. However, this model does not considering the thermal resistance across the carbon nanotube–fluid interface. Later, Nan et al. [96] modified their model and tried to describe the effect of the interface thermal resistance. However, the model still cannot explain the nonlinear phenomena of the effective thermal conductivity of nanotube suspensions with nanotube volume fractions. Recently, to account for the geometric and physical anisotropy simultaneously, Gao and Zhou [97] proposed a differential effective medium theory based on Bruggeman's model [80] to predict the effective thermal conductivity of nanofluids. Although their model involves the effect of aspect ratio of the nanotube, the size effect and temperature dependence have not be included. From the results, the prediction of the thermal conductivity of normal nanofluids rather than nanotube-based suspensions is not good.

Recently, Xue [98] proposed a Maxwell theory based model of the effective thermal conductivity of CNTs-nanofluids to include the effect of large axial ratio and the space distribution of the CNTs. As compared with the existing experimental data [32], the proposed model provided reasonable agreement with adjusted thermal conductivity of CNTs. With the assumed distribution function $P(B_j) = 2$, the corresponding expression of the effective thermal conductivity of CNTs-based nanofluids is

$$k_{\text{eff}} = k_{\text{b}}\frac{1 - \phi + 2\phi\frac{k_{\text{p}}}{k_{\text{p}} - k_{\text{b}}}\ln\frac{k_{\text{p}} + k_{\text{b}}}{2k_{\text{b}}}}{1 - \phi + 2\phi\frac{k_{\text{b}}}{k_{\text{p}} - k_{\text{b}}}\ln\frac{k_{\text{p}} + k_{\text{b}}}{2k_{\text{b}}}} \tag{18}$$

Table 5
Analytical models on thermal conductivity of nanofluids – a summary

Investigator	Formula ($k_{\text{eff}}/k_{\text{b}}$)	Comments
Maxwell [1]	$\frac{k_{\text{p}}+2k_{\text{b}}+2(k_{\text{p}}-k_{\text{b}})\phi}{k_{\text{p}}+2k_{\text{b}}-(k_{\text{p}}-k_{\text{b}})\phi}$	relates the thermal conductivity of spherical particle, base fluid and solid volume fraction
Hamilton and Crosser [27]	$\frac{k_{\text{p}}+(n-1)k_{\text{b}}-(n-1)(k_{\text{b}}-k_{\text{p}})\phi}{k_{\text{p}}+(n-1)k_{\text{b}}+(k_{\text{b}}-k_{\text{p}})\phi}$	for non-spherical particles, $k_{\text{p}}/k_{\text{b}} > 100$, n is an empirical shape factor ($n = 3/\psi$, ψ is the sphericity)
Wasp [100]	$\frac{k_{\text{p}}+2k_{\text{b}}-2(k_{\text{b}}-k_{\text{p}})\phi}{k_{\text{p}}+2k_{\text{b}}+(k_{\text{b}}-k_{\text{p}})\phi}$	special case of Hamilton and Crosser's model with $\psi = 1$
Bruggeman [80]	$\frac{1}{4}[(3\phi-1)\frac{k_{\text{p}}}{k_{\text{b}}} + (2-3\phi) + \frac{1}{4}\sqrt{\Delta}]$	$\Delta = [(3\phi-1)^2(k_{\text{p}}/k_{\text{b}})^2 + (2-3\phi)^2 + 2(2+9\phi-9\phi^2)(k_{\text{p}}/k_{\text{b}})]$
Yu and Choi [70,71]	$\frac{k_{\text{pe}}+2k_{\text{b}}+2(k_{\text{pe}}-k_{\text{b}})(1-\beta)^3\phi}{k_{\text{pe}}+2k_{\text{b}}-(k_{\text{pe}}-k_{\text{b}})(1+\beta)^3\phi}$, $1+\frac{n\phi_{\text{eff}}A}{1-\phi_{\text{eff}}A}$	a modified Maxwell and Hamilton–Crosser model, respectively
Wang et al. [88]	$\frac{(1-\phi)+3\phi\int_0^\infty k_{\text{cl}}(r)n(r)/[k_{\text{cl}}(r)+2k_{\text{b}}]\,\text{d}r}{(1-\phi)+3\phi\int_0^\infty k_{\text{b}}(r)n(r)/[k_{\text{cl}}(r)+2k_{\text{b}}]\,\text{d}r}$	a fractal model based on effective medium approximation and fractal theory
Xue [82]	$0 = 9(1-\frac{\phi}{\lambda})\frac{k_{\text{eff}}-k_{\text{b}}}{2k_{\text{eff}}+k_{\text{b}}} + \frac{\phi}{\lambda}[\frac{k_{\text{eff}}-k_{\text{c},x}}{k_{\text{eff}}+B_{2,x}(k_{\text{c},x}-k_{\text{eff}})} + 4\frac{k_{\text{eff}}-k_{\text{c},y}}{2k_{\text{eff}}+(1-B_{2,x})(k_{\text{c},y}-k_{\text{eff}})}]$	includes the effect of interface between solid particles and base fluid
Xue and Xu [84]	$0 = (1-\frac{\phi}{\alpha})\frac{k_{\text{eff}}-k_{\text{b}}}{2k_{\text{eff}}+k_{\text{b}}} + \frac{\phi}{\alpha}\frac{(k_{\text{eff}}-k_2)(2k_2+k_1)-\alpha(k_1-k_2)(2k_2+k_{\text{eff}})}{(2k_{\text{eff}}+k_2)(2k_2+k_1)+2\alpha(k_1-k_2)(k_2-k_{\text{eff}})}$	a modified Bruggeman model including effect of interfacial shells
Xie et al. [85]	$1+3\Theta\phi_{\text{T}} + \frac{2\Theta^2\phi_{\text{T}}^2}{1-\Theta\phi_{\text{T}}}$	includes effect of nanolayer
Xuan et al. [86]	$\frac{k_{\text{p}}+2k_{\text{b}}-2(k_{\text{b}}-k_{\text{p}})\phi}{k_{\text{p}}+2k_{\text{b}}+(k_{\text{b}}-k_{\text{p}})\phi} + \frac{\rho_{\text{p}}\phi c_{\text{p}}}{2k_{\text{b}}}\sqrt{\frac{k_{\text{B}}T}{3\pi r_{\text{c}}\eta}}$	includes effect of the random motion of the suspended nanoparticles as well as the interfacial interactions
Kumar et al. [89]	$1+c\frac{2k_{\text{B}}T}{(\pi\eta d_{\text{p}}^2)}\frac{\varepsilon r_{\text{b}}}{k_{\text{b}}(1-\varepsilon)r_{\text{p}}}$	particle size, concentration, and temperature
Bhattacharya et al. [90]	$\frac{k_{\text{p}}}{k_{\text{b}}}\phi + (1-\phi)$	$k_{\text{p}} = \frac{1}{k_{\text{B}}T^2V}\sum_{j=0}^{n}\langle Q(0)Q(j\Delta t)\rangle\Delta t$
Jang and Choi [91]	$k_{\text{b}}(1-\phi) + k_{\text{p}}\phi + 3C\frac{d_{\text{b}}}{d_{\text{p}}}k_{\text{b}}Re_{d_{\text{p}}}^2 Pr\phi$	four modes: collisons between fluid molecules, thermal diffusion of nanoparticles, collisons between nanoparticles due to Brownian motion and thermal interaction of dynamic nanoparticles with base fluid molecules
Prasher [92]	$(1+A\,Re^m\,Pr^{0.333}\phi)[\frac{k_{\text{p}}+2k_{\text{b}}+2(k_{\text{p}}-k_{\text{b}})\phi}{k_{\text{p}}+2k_{\text{b}}-(k_{\text{p}}-k_{\text{b}})\phi}]$	effect of convection of the liquid near the particle included
Koo and Kleinstreuer [93,94]	$\frac{k_{\text{eff,Maxwell}}}{k_{\text{b}}} + 5\times10^4\beta\phi\rho_{\text{p}}c_{\text{p}}\sqrt{\frac{\kappa T}{\rho_{\text{p}}D}}\frac{f(T,\phi)}{k_{\text{b}}}$	surrounding liquid motion with randomly moving nanoparticles considered
Xue [98]	$\frac{1-\phi+2\phi\frac{k_{\text{p}}}{k_{\text{p}}-k_{\text{b}}}\ln\frac{k_{\text{p}}+k_{\text{b}}}{2k_{\text{b}}}}{1-\phi+2\phi\frac{k_{\text{b}}}{k_{\text{p}}-k_{\text{b}}}\ln\frac{k_{\text{p}}+k_{\text{b}}}{2k_{\text{b}}}}$	for CNTs-based nanofluids and including the axial ratio and the space distribution

Xue [99] also presented a model of the effective thermal conductivity for carbon nanotube composites by incorporating the interface thermal resistance with an average polarization theory. The proposed model includes the effects of carbon nanotube length, diameter, concentration, interface thermal resistance, and the thermal conductivities of nanotube and base fluid on the thermal conductivity of the nanofluid simultaneously.

$$9(1-\phi)\frac{k_{\text{eff}}-k_{\text{b}}}{2k_{\text{eff}}+k_{\text{b}}} + \phi\left[\frac{k_{\text{eff}}-k_{33}^c}{k_{\text{eff}}+0.14\frac{d}{L}(k_{33}^c-k_{\text{eff}})} + 4\frac{k_{\text{eff}}-k_{11}^c}{2k_{\text{eff}}+\frac{1}{2}(k_{11}^c-k_{\text{eff}})}\right] = 0 \tag{19}$$

where k_{11}^c and k_{33}^3 are the transverse and longitudinal equivalent thermal conductivities of the composite unit cell of a nanotube with length L and diameter d. The model predicts that the nanotube diameter has a neglected effect on the thermal conductivity enhancement of the nanotube nanofluids. It seems that the model can agree well with the data from Xia et al. [33].

A summary of selective analytical models on thermal conductivity of nanofluids is listed in Table 5. Fig. 4 shows comparison between selected discussed theoretical models and experimental data on thermal conductivity for Al_2O_3/water nanofluids. To obtain deeper understanding of the effective thermal conductivity of nanofluids, some important facts must be taken to account in future studies. Such facts include effect of the size and shape of the nanoparticles, the interfacial contact resistance between nanoparticles and base fluids, the temperature dependence or the effect of Brownian motion, and the effect of clustering of particles.

4.3. Numerical investigations

For numerical simulations two approaches have been adopted in the literature to investigate the heat transfer characteristics of nanofluids. The first approach assumes that the continuum assumption is still valid for fluids with suspended nanosize particles. The other approach uses a two-phase model for better description of both the fluid and the solid phases, but it

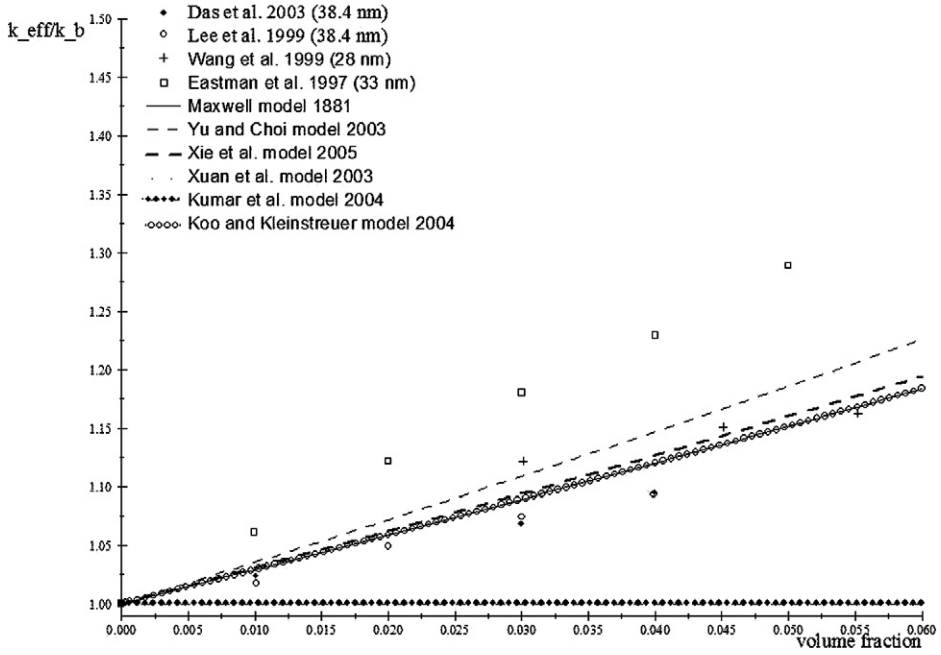

Fig. 4. Comparison between selected theoretical models and experimental data on thermal conductivity for Al_2O_3/water nanofluids.

is not common in the open literature. The single phase model is much simpler and computationally more efficient. Another approach is to adopt the Boltzmann theory. The heat transfer enhancement using nanofluids may be affected by several factors such as the Brownian motion, layering at the solid/liquid interface, ballistic phonon transport through the particles, nanoparticle clustering and the friction between the fluid and the solid particles. It is difficult to describe all these phenomena mathematically, however.

Maïga et al. [101,102] numerically investigated the hydrodynamic and thermal characteristics of nanofluids flowing through a uniformly heated tube ($L = 1$ m) in both laminar and turbulent regimes using the single phase model with adjusted properties. Results showed that the addition of nanoparticles can increase the heat transfer substantially compared to the base fluid alone. It was also found that the ethylene glycol–γAl_2O_3 provided better heat transfer enhancement than the water–γAl_2O_3 nanofluids. However, Maïga et al. [103] also discussed the disadvantages of nanofluids with respect to heat transfer. The inclusion of nanoparticles introduced drastic effects on the wall shear stress, which increased with increase of solid volume fraction. A new correlation was proposed by Maïga et al. [104] to describe the thermal performance of Al_2O_3/water nanofluids under turbulent regime, $Nu_{\text{fully}} = 0.085\,Re^{0.71}\,Pr^{0.35}$, which is valid for $10^4 \leqslant Re \leqslant 5 \times 10^5$, $6.6 \leqslant Pr \leqslant 13.9$ and $0 \leqslant \phi \leqslant 10$.

Roy et al. [105] conducted a numerical study of heat transfer for water–γAl_2O_3 nanofluids in a radial cooling system. They found that addition of nanoparticles in the base fluids increased the heat transfer rates considerably. Use of 10 vol% nanoparticles resulted in a two-fold increase of the heat transfer rate as compared to that of the pure base fluid. Their results are similar to those of Maïga et al. [101,102] since they both used the same model.

Wang et al. [106] investigated numerically free convective heat transfer characteristics of a two-dimensional cavity over a range Grashof numbers and solid volume fractions for various nanofluids. Their results showed that suspended nanoparticles significantly increased the heat transfer rate at all Grashof numbers. For water–γAl_2O_3 nanofluid, the increase of the average heat transfer coefficient was approximately 30% for 10 vol% nanoparticles. The maximum increase in heat transfer performance of 80% was obtained for 10 vol% Cu nanoparticles dispersed in water. Furthermore, the average heat transfer coefficient was seen to increase by up to 100% for the nanofluid consisting of oil containing 1 vol% carbon nanotubes. Furthermore, the presence of nanoparticles in the base fluid was found to alter the structure of the fluid flow for horizontal orientation of the heated wall.

Khanafer et al. [54] developed an analytical model to determine natural convective heat transfer in nanofluids. The nanofluid in the enclosure was assumed to be single phase. The effect of suspended nanoparticles on buoyancy-driven heat transfer process was analyzed. It was observed that the heat transfer rate increased as the particle volume fraction increased at any given Grashof number. Kim et al. [83] analytically investigated the instability in natural convection of nanofluids by introducing a new factor (f), which measures the ratio of the Rayleigh number of the nanofluids to that of the base fluids, to describe the effect of nanoparticle addition on the convective instability and heat transport of the base fluid. Results demonstrated that the increased particle volume fraction improves the heat transfer rates in nanofluids compared to those in the base fluid alone.

Xuan and Roetzel [107] derived several correlations for convective heat transfer for nanofluids. Both single phase and two phase models were used to explain the mechanism of the increased heat transfer rates. However, there are few experimental

data to validate such models. Jang and Choi [108] investigated the natural stability of water-based nanofluids containing 6 nm copper and 2 nm diamond nanoparticles in a rectangular cavity heated from the bottom. They noted that nanofluids were more stable compared to the base fluids.

From the microscopic point of view, the traditional computational methods for two-phase mixture flow do not reveal the inherent nature of the fluid flow and heat transfer characteristics of nanofluids. A microscopic approach needs to be introduced to describe the effects of interactions between the suspended nanoparticles and the base liquid particles as well as among the solid particles. The lattice Boltzmann equation is one of the methods available to deal with such problems. By considering the external and internal forces on the nanoparticles and the mechanical and thermal interactions among the nanoparticle and fluid molecules, Xuan and Yao [109] simulated nanoparticle distributions and flow of nanofluids using the lattice Boltzmann model. The increased temperature of the fluids could increase the nanoparticle distribution, which is an important factor responsible for heat transfer enhancement in nanofluids. Xuan et al. [110] observed that the random motion of nanoparticles tends to flatten the temperature distribution near the boundary wall. Due to the irregular fluctuation of suspended nanoparticles, the Nusselt distribution would fluctuate along the main flow direction rather than the smooth distribution from the base fluid. Their results indicated that the distribution and volume fraction of the nanoparticles were important factors determining the temperature distribution and heat transfer improvement with nanofluids.

Another interesting numerical investigation was conducted by Xue et al. [72] using non-equilibrium molecular dynamics simulations. They studied the effect of the liquid–solid interface on the interfacial thermal resistance and found that the simple monatomic liquid around the solid particle showed no influence on the thermal transport either normal to the surface or parallel to the surface. They suggested that the large improvement of thermal conductivity in nanofluids cannot be explained by the thermal transport in the liquid–solid interface layer.

In summary, it is difficult to identify an established theory to predict accurately the heat transfer characteristics of nanofluids. Many researchers deal with the nanofluids as single-phase fluid rather than two-phase mixture. However, the particle–liquid interaction and the movement between the particle and liquids should play important roles in affecting the convective heat transfer performance of nanofluids.

5. Conclusions

This paper presents an overview of the recent developments in the study on heat transfer using nanofluids. Many important, complex and interesting phenomena involving nanofluids have been reported in the literature. Researchers investigate have given much attention on the thermal conductivity rather than the heat transfer characteristics. The use of nanofluids in a wide range of applications appears promising, but the development of the field faces several challenges: (i) the lack of agreement between experimental results from different groups; (ii) the often poor performance of suspensions; and lack of theoretical understanding of the mechanisms. Further theoretical and experimental research investigations are needed to understand the heat transfer characteristics of nanofluids and identify new and unique applications for these fields.

References

[1] J.C. Maxwell, A Treatise on Electricity and Magnetism, second ed., Clarendon Press, Oxford, UK, 1881.
[2] S.U.S. Choi, Enhancing thermal conductivity of fluids with nanoparticles, Developments and Applications of Non-Newtonian Flows, FED-vol. 231/MD-vol. 66, 1995, pp. 99–105.
[3] P. Keblinski, J.A. Eastman, D.G. Cahill, Nanofluids for thermal transport, Materials Today 8 (6) (2005) 36–44.
[4] H. Akoh, Y. Tsukasaki, S. Yatsuya, A. Tasaki, Magnetic properties of ferromagnetic ultrafine particles prepared by vacuum evaporation on running oil substrate, Journal of Crystal Growth 45 (1978) 495–500.
[5] M. Wagener, B.S. Murty, B. Gunther, Preparation of metal nanosuspensions by high-pressure DC-sputtering on running liquids, in: S. Komarnenl, J.C. Parker, H.J. Wollenberger (Eds.), Nanocrystalline and Nanocomposite Materials II, vol. 457, Materials Research Society, Pittsburgh, PA, 1997, pp. 149–154.
[6] J.A. Eastman, U.S. Choi, S. Li, L.J. Thompson, S. Lee, Enhanced thermal conductivity through the development of nanofluids, Materials Research Society Symposium – Proceedings, vol. 457, Materials Research Society, Pittsburgh, PA, USA, Boston, MA, USA, 1997, pp. 3–11.
[7] H. Zhu, Y. Lin, Y. Yin, A novel one-step chemical method for preparation of copper nanofluids, Journal of Colloid and Interface Science 227 (2004) 100–103.
[8] C.-H. Lo, T.-T. Tsung, L.-C. Chen, Shape-controlled synthesis of Cu-based nanofluid using submerged arc nanoparticle synthesis system (SANSS), Journal of Crystal Growth 277 (1–4) (2005) 636–642.
[9] C.-H. Lo, T.-T. Tsung, L.-C. Chen, Ni nano-magnetic fluid prepared by submerged arc nano synthesis system (sanss), JSME International Journal, Series B: Fluids and Thermal Engineering 48 (4) (2006) 750–755.
[10] S. Lee, S.U.S. Choi, S. Li, J.A. Eastman, Measuring thermal conductivity of fluids containing oxide nanoparticles, Journal of Heat Transfer 121 (1999) 280–289.
[11] X. Wang, X. Xu, S.U.S. Choi, Thermal conductivity of nanoparticle–fluid mixture, Journal of Thermophysics and Heat Transfer 13 (4) (1999) 474–480.
[12] S.M.S. Murshed, K.C. Leong, C. Yang, Enhanced thermal conductivity of TiO_2–water based nanofluids, International Journal of Thermal Sciences 44 (4) (2005) 367–373.
[13] Y. Xuan, Q. Li, Heat transfer enhancement of nanofluids, International Journal of Heat and Fluid Transfer 21 (2000) 58–64.
[14] Y.J. Hwang, Y.C. Ahn, H.S. Shin, C.G. Lee, G.T. Kim, H.S. Park, J.K. Lee, Investigation on characteristics of thermal conductivity enhancement of nanofluids, Current Applied Physics, in press.
[15] J. Kestin, W.A. Wakeham, A contribution to the theory of the transient hot-wire technique for thermal conductivity measurements, Physica A 92 (1978) 102–116.
[16] S.K. Das, N. Putta, P. Thiesen, W. Roetzel, Temperature dependence of thermal conductivity enhancement for nanofluids, ASME Trans. J. Heat Transfer 125 (2003) 567–574.
[17] Y. Nagasaka, A. Nagashima, Absolute measurement of the thermal conductivity of electrically conducting liquids by the transient hot-wire method, Journal of Physics E: Scientific Instruments 14 (1981) 1435–1440.
[18] W. Roetzel, S. Prinzen, Y. Xuan, Measurement of thermal diffusivity using temperature oscillations, in: C. Cremers, H. Fine (Eds.), Thermal Conductivity, vol. 21, Plenum Press, New York and London, 1990, pp. 201–207.
[19] W. Czarnetzki, W. Roetzel, Temperature oscillation techniques for simultaneous measurement of thermal diffusivity and conductivity, International Journal of Thermophysics 16 (2) (1995) 413–422.

[20] H. Masuda, A. Ebata, K. Teramae, N. Hishinuma, Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles (dispersion of 7-AI2O3, SiO_2, and TiO_2 ultra-fine particles), Netsu Bus-sei (Japan) 7 (4) (1993) 227–233.

[21] H. Xie, J. Wang, T. Xi, Y. Liu, F. Ai, Q. Wu, Thermal conductivity enhancement of suspensions containing nanosized alumina particles, Journal of Applied Physics 91 (7) (2002) 4568–4572.

[22] J.A. Eastman, S.U.S. Choi, S. Li, W. Yu, L.J. Thompson, Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles, Applied Physics Letters 78 (6) (2001) 718–720.

[23] T.-K. Hong, H.-S. Yang, C.J. Choi, Study of the enhanced thermal conductivity of Fe nanofluids, Journal of Applied Physics 97 (6) (2005) 1–4.

[24] K. Hong, T.-K. Hong, H.-S. Yang, Thermal conductivity of fe nanofluids depending on the cluster size of nanoparticles, Applied Physics Letters 88 (3) (2006) 31901.

[25] H. Xie, J. Wang, T. Xi, Y. Liu, Study on the thermal conductivity of sic nanofluids, Journal of the Chinese Ceramic Society 29 (4) (2001) 361–364.

[26] H. Xie, J. Wang, T. Xi, Y. Liu, Thermal conductivity of suspensions containing nanosized SiC particles, International Journal of Thermophysics 23 (2) (2002) 571–580.

[27] R.L. Hamilton, O.K. Crosser, Thermal conductivity of heterogeneous two-component systems, I&EC Fundam 1 (1962) 182–191.

[28] C.H. Li, G.P. Peterson, Experimental investigation of temperature and volume fraction variations on the effective thermal conductivity of nanoparticle suspensions (nanofluids), Journal of Applied Physics 99 (8) (2006) 084314.

[29] H.E. Patel, S.K. Das, T. Sundararagan, A.S. Nair, B. Geoge, T. Pradeep, Thermal conductivities of naked and monolayer protected metal nanoparticle based nanofluids: Manifestation of anomalous enhancement and chemical effects, Applied Physics Letters 83 (2003) 2931–2933.

[30] S.A. Putnam, D.G. Cahill, P.V. Braun, Z. Ge, R.G. Shimmin, Thermal conductivity of nanoparticle suspensions, Journal of Applied Physics 99 (8) (2006) 084308.

[31] S. Iijima, Helical microtubules of graphitic carbon, Nature 354 (6348) (1991) 56–57.

[32] S.U.S. Choi, Z.G. Zhang, W. Yu, F.E. Lockwood, E.A. Grulke, Anomalous thermal conductivity enhancement in nano-tube suspensions, Applied Physics Letters 79 (2001) 2252–2254.

[33] H. Xie, H. Lee, W. Youn, M. Choi, Nanofluids containing multiwalled carbon nanotubes and their enhanced thermal conductivities, Journal of Applied Physics 94 (8) (2003) 4967–4971.

[34] M. Biercuk, M. Llaguno, M. Radosavljevic, J. Hyun, A. Johnson, J. Fischer, Carbon nanotube composites for thermal management, Applied Physics Letters 80 (15) (2002) 2767–2769.

[35] M. Llaguno, J. Hone, A. Johnson, J. Fischer, Thermal conductivity of single-wall carbon nanotubes: diameter and annealing dependence, in: H. Kuzmany, J. Fink, M. Mehring, S. Roth (Eds.), AIP Conference Proceedings, vol. 591, Woodbury, New York, 2001.

[36] E.S. Choi, J.S. Brooks, D.L. Eaton, M.S. Al-Haik, M.Y. Hussaini, H. Garmestani, D. Li, K. Dahmen, Enhancement of thermal and electrical properties of carbon nanotube polymer composites by magnetic field processing, Journal of Applied Physics 94 (9) (2003) 6034–6039.

[37] D. Wen, Y. Ding, Effective thermal conductivity of aqueous suspensions of carbon nanotubes (carbon nanotube nanofluids), Journal of Thermophysics and Heat Transfer 18 (4) (2004) 481–485.

[38] Y. Ding, H. Alias, D. Wen, R.A. Williams, Heat transfer of aqueous suspensions of carbon nanotubes (CNT nanofluids), International Journal of Heat and Mass Transfer 49 (1–2) (2005) 240–250.

[39] M.J. Assael, C.F. Chen, I.N. Metaxa, W.A. Wakeham, Thermal conductivity of suspensions of carbon nanotubes in water, in: 15th Symposium on Thermophysical Properties, National Institute of Standards, University of Colorado, Boulder, USA, 2003.

[40] M.J. Assael, C.F. Chen, I.N. Metaxa, W.A. Wakeham, Thermal conductivity of suspensions of carbon nanotubes in water, International Journal of Thermophysics 25 (4) (2004) 971–985.

[41] M.J. Assael, I.N. Metaxa, J. Arvanitidis, D. Christofilos, C. Lioutas, Thermal conductivity enhancement in aqueous suspensions of carbon multi-walled and double-walled nanotubes in the presence of two different dispersants, International Journal of Thermophysics 26 (3) (2005) 647–664.

[42] M.-S. Liu, M. Ching-Cheng Lin, I.T. Huang, C.-C. Wang, Enhancement of thermal conductivity with carbon nanotube for nanofluids, International Communications in Heat and Mass Transfer 32 (9) (2005) 1202–1210.

[43] J.M. Li, Z.L. Li, B.X. Wang, Experimental viscosity measurements for copper oxide nanoparticle suspensions, Tsinghua Sci. Tech. 7 (2) (2002) 198–201.

[44] S.K. Das, N. Putra, W. Roetzel, Pool boiling characteristics of nano-fluids, International Journal of Heat and Mass Transfer 46 (5) (2003) 851–862.

[45] S. Lee, S.U.S. Choi, Application of metallic nanoparticle suspensions in advanced cooling systems, in: 1996 International Mechanical Engineering Congress and Exhibition, Atlanta, USA, 1996.

[46] Y. Xuan, Q. Li, Investigation on convective heat transfer and flow features of nanofluids, Journal of Heat Transfer 125 (2003) 151–155.

[47] D. Wen, Y. Ding, Experimental investigation into convective heat transfer of nanofluids at the entrance region under laminar flow conditions, International Journal of Heat and Mass Transfer 47 (24) (2004) 5181.

[48] S. Heris, S.G. Etemad, M. Esfahany, Experimental investigation of oxide nanofluids laminar flow convective heat transfer, International Communications in Heat and Mass Transfer 33 (4) (2006) 529–535.

[49] H.-T. Chien, C.-I. Tsai, P.-H. Chen, P.-Y. Chen, Improvement on thermal performance of a disk-shaped miniature heat pipe with nanofluid, in: ICEPT 2003, Fifth International Conference on Electronic Packaging Technology, Proceedings (IEEE Cat. No.03EX750), IEEE, Shanghai, China, 2003, p. 389.

[50] C.Y. Tsai, H.T. Chien, P.P. Ding, B. Chan, T.Y. Luh, P.H. Chen, Effect of structural character of gold nanoparticles in nanofluid on heat pipe thermal performance, Material Letters 58 (2004) 1461–1465.

[51] H.B. Ma, C. Wilson, B. Borgmeyer, K. Park, Q. Yu, S.U.S. Choi, M. Tirumala, Effect of nanofluid on the heat transport capability in an oscillating heat pipe, Applied Physics Letters 88 (14) (2006) 143116.

[52] B. Pak, Y. Cho, Hydrodynamic and heat transfer study of dispersed fluids with submicron metallic oxide particles, Experimental Heat Transfer 11 (2) (1998) 151–170.

[53] Y. Yang, Z.G. Zhang, E.A. Grulke, W.B. Anderson, G. Wu, Heat transfer properties of nanoparticle-in-fluid dispersions (nanofluids) in laminar flow, International Journal of Heat and Mass Transfer 48 (6) (2005) 1107–1116.

[54] K. Khanafer, K. Vafai, M. Lightstone, Bouyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids, International Journal of Heat and Mass Transfer 46 (2003) 3639–3653.

[55] N. Putra, W. Roetzel, S.K. Das, Natural convection of nano-fluids, Heat and Mass Transfer 39 (8–9) (2003) 775–784.

[56] D. Wen, Y. Ding, Formulation of nanofluids for natural convective heat transfer applications, International Journal of Heat and Fluid Flow 26 (6) (2005) 855–864.

[57] D. Faulkner, M. Khotan, R. Shekarriz, Practical design of a 1000 W/cm^2 cooling system, in: Annual IEEE Semiconductor Thermal Measurement and Management Symposium, Institute of Electrical and Electronics Engineers Inc., San Jose, CA, United States, 2003, pp. 223–230.

[58] S. Witharana, Boiling of refrigerants on enhanced surfaces and boiling of nanofluids, PhD thesis, The Royal Institute of Technology, 2003.

[59] C.H. Li, B.X. Wang, X.F. Peng, Experimental investigations on boiling of nano-particle suspensions, in: 2003 Boiling Heat Transfer Conference, Jamica, USA, 2003.

[60] S.K. Das, N. Putra, W. Roetzel, Pool boiling of nano-fluids on horizontal narrow tubes, International Journal of Multiphase Flow 29 (8) (2003) 1237–1247.

[61] I.C. Bang, S.H. Chang, Boiling heat transfer performance and phenomena of Al_2O_3–water nanofluids from a plain surface in a pool, in: Proceedings of ICAPP, Pitterburgh, US, 2004.

[62] I.C. Bang, S.H. Chang, Boiling heat transfer performance and phenomena of Al_2O_3–water nano-fluids from a plain surface in a pool, International Journal of Heat and Mass Transfer 48 (12) (2005) 2420–2428.

[63] J.P. Tu, N. Dinh, T. Theofanous, An experimental study of nanofluid boiling heat transfer, in: Proceedings of 6th International Symposium on Heat Transfer, Beijing, China, 2004.
[64] S.M. You, J.H. Kim, K.H. Kim, Effect of nanoparticles on critical heat flux of water in pool boiling heat transfer, Applied Physics Letters 83 (2003) 3374–3376.
[65] P. Vassallo, R. Kumar, S.D. Amico, Pool boiling heat transfer experiments in silica–water nano-fluids, International Journal of Heat and Mass Transfer 47 (2004) 407–411.
[66] D.W. Zhou, Heat transfer enhancement of copper nanofluid with acoustic cavitation, International Journal of Heat and Mass Transfer 47 (2004) 3109–3117.
[67] D. Wen, Y. Ding, Experimental investigation into the pool boiling heat transfer of aqueous based γ-alumina nanofluids, Journal of Nanoparticle Research 7 (2) (2005) 265–274.
[68] P. Keblinski, S.R. Phillpot, S.U.S. Choi, J.A. Eastman, Mechanisms of heat flow in suspensions of nano-sized particles (nanofluids), International Journal of Heat and Mass Transfer 45 (2002) 855–863.
[69] J.A. Eastman, S.R. Phillpot, S.U.S. Choi, P. Keblinski, Thermal transport in nanofluids, Annu. Rev. Mater. Res. 34 (2004) 219–246.
[70] W. Yu, S.U.S. Choi, The role of interfacial layers in the enhanced thermal of nanofluids: a renovated Maxwell model, Journal of Nanoparticle Research 5 (1–2) (2003) 167–171.
[71] W. Yu, S.U.S. Choi, The role of interfacial layers in the enhanced thermal conductivity of nanofluids: a renovated Hamilton–Crosser model, Journal of Nanoparticle Research 6 (4) (2004) 355–361.
[72] L. Xue, P. Keblinski, S.R. Phillpot, S.U.S. Choi, J.A. Eastman, Effect of liquid layering at the liquid–solid interface on thermal transport, International Journal of Heat and Mass Transfer 47 (19–20) (2004) 4277–4284.
[73] A.R.A. Khaled, K. Vafai, Heat transfer enhancement through control of thermal dispersion effects, International Journal of Heat and Mass Transfer 48 (11) (2005) 2172.
[74] D. Wen, Y. Ding, Effect of particle migration on heat transfer in suspensions of nanoparticles flowing through minichannels, Microfluidics and Nanofluidics 1 (2) (2005) 183–189.
[75] Y. Ding, D. Wen, Particle migration in a flow of nanoparticle suspensions, Powder Technology 149 (2–3) (2005) 84–92.
[76] J. Koo, C. Kleinstreuer, Impact analysis of nanoparticle motion mechanisms on the thermal conductivity of nanofluids, International Communications in Heat and Mass Transfer 32 (9) (2005) 1111–1118.
[77] W. Evans, J. Fish, P. Keblinski, Role of Brownian motion hydrodynamics on nanofluid thermal conductivity, Applied Physics Letters 88 (9) (2006) 93116.
[78] D. Lee, J.-W. Kim, B.G. Kim, A new parameter to control heat transport in nanofluids: Surface charge state of the particle in suspension, Journal of Physical Chemistry B 110 (9) (2006) 4323–4328.
[79] P. Vadasz, Heat conduction in nanofluid suspensions, Transactions of the ASME, Journal of Heat Transfer 128 (5) (2006) 465–477.
[80] D.A.G. Bruggeman, Berechnung verschiedener physikalischer Konstanten von heterogenen Substanzen, I. Dielektrizitatskonstanten und Leitfahigkeiten der Mischkorper aus isotropen Substanzen, Annalen der Physik, Leipzig 24 (1935) 636–679.
[81] L. Schwartz, E. Garboczi, D. Bentz, Interfacial transport in porous media: Application to dc electrical conductivity of mortars, Journal of Applied Physics 78 (10) (1995) 5898–5908.
[82] Q.-Z. Xue, Model for effective thermal conductivity of nanofluids, Physics Letters A 307 (2003) 313–317.
[83] J. Kim, Y.T. Kang, C.K. Choi, Analysis of convective instability and heat transfer characteristics of nanofluids, Physics of Fluids 16 (7) (2004) 2395–2401.
[84] Q. Xue, W.-M. Xu, A model of thermal conductivity of nanofluids with interfacial shells, Materials Chemistry and Physics 90 (2–3) (2005) 298–301.
[85] H. Xie, M. Fujii, X. Zhang, Effect of interfacial nanolayer on the effective thermal conductivity of nanoparticle–fluid mixture, International Journal of Heat and Mass Transfer 48 (14) (2005) 2926–2932.
[86] Y. Xuan, Q. Li, W. Hu, Aggregation structure and thermal conductivity of nanofluids, AIChE Journal 49 (4) (2003) 1038–1043.
[87] B.B. Mandelbrot, The Fractal Geometry of Nature, W.H. Freeman Press, San Francisco, 1982.
[88] B.-X. Wang, L.-P. Zhou, X.-F. Peng, A fractal model for predicting the effective thermal conductivity of liquid with suspension of nanoparticles, International Journal of Heat and Mass Transfer 46 (2003) 2665–2672.
[89] D.H. Kumar, H.E. Patel, V.R.R. Kumar, T. Sundararajan, T. Pradeep, S.K. Das, Model for heat conduction in nanofluids, Physical Review Letters 93 (14) (2004) 144301.
[90] P. Bhattacharya, S.K. Saha, A. Yadav, P.E. Phelan, R.S. Prasher, Brownian dynamics simulation to determine the effective thermal conductivity of nanofluids, Journal of Applied Physics 95 (11) (2004) 6492–6494.
[91] S.P. Jang, S.U.S. Choi, Role of Brownian motion in the enhanced thermal conductivity of nanofluids, Applied Physics Letters 84 (2004) 4316–4318.
[92] R. Prasher, P. Bhattacharya, P.E. Phelan, Thermal conductivity of nanoscale colloidal solutions (nanofluids), Physical Review Letters 94 (2) (2005) 025901.
[93] J. Koo, C. Kleinstreuer, A new thermal conductivity model for nanofluids, Journal of Nanoparticle Research 6 (6) (2004) 577–588.
[94] J. Koo, C. Kleinstreuer, Laminar nanofluid flow in micro-heat sinks, International Journal of Heat and Mass Transfer 48 (13) (2005) 2652–2661.
[95] C.-W. Nan, Z. Shi, Y. Lin, A simple model for thermal conductivity of carbon nanotube-based composites, Chemical Physics Letters 375 (5–6) (2003) 666–669.
[96] C.-W. Nan, G. Liu, Y. Lin, M. Li, Interface effect on thermal conductivity of carbon nanotube composites, Applied Physics Letters 85 (2004) 3549–3551.
[97] L. Gao, X.F. Zhou, Differential effective medium theory for thermal conductivity in nanofluids, Physics Letters A, in press.
[98] Q.Z. Xue, Model for thermal conductivity of carbon nanotube-based composites, Physica B: Condensed Matter 368 (1–4) (2005) 302–307.
[99] Q. Xue, Model for the effective thermal conductivity of carbon nanotube composites, Nanotechnology 17 (6) (2006) 1655–1660.
[100] E.J. Wasp, J.P. Kenny, R.L. Gandhi, Solid–liquid Slurry Pipeline Transportation, Bulk Materials Handling, Trans Tech Publications, Germany, 1999.
[101] S.E.B. Maiga, C.T. Nguyen, N. Galanis, G. Roy, Heat transfer behaviours of nanofluids in a uniformly heated tube, Superlattices and Microstructures 35 (2004) 543–557.
[102] S.E.B. Maiga, C.T. Nguyen, N. Galanis, G. Roy, Hydrodynamic and thermal behaviours of a nanofluid in a uniformly heated tube, in: Computational Studies, vol. 5, WIT Press, Southampton, SO40 7AA, United Kingdom, Lisbon, Portugal, 2004, pp. 453–462.
[103] S.E.B. Maiga, S.J. Palm, C.T. Nguyen, G. Roy, N. Galanis, Heat transfer enhancement by using nanofluids in forced convection flows, International Journal of Heat and Fluid Flow 26 (4) (2005) 530–546.
[104] S.E.B. Maiga, N. Cong Tam, N. Galanis, G. Roy, T. Mare, M. Coqueux, Heat transfer enhancement in turbulent tube flow using Al_2O_3 nanoparticle suspension, International Journal of Numerical Methods for Heat and Fluid Flow 16 (3) (2006) 275–292.
[105] G. Roy, C.T. Nguyen, P.-R. Lajoie, Numerical investigation of laminar flow and heat transfer in a radial flow cooling system with the use of nanofluids, Superlattices and Microstructures 35 (2004) 497–511.
[106] X.-Q. Wang, A.S. Mujumdar, C. Yap, Free convection heat transfer in horizontal and vertical rectangular cavities filled with nanofluids, in: International Heat Transfer Conference IHTC-13, Sydney, Australia, August, 2006.
[107] Y. Xuan, W. Roetzel, Conceptions for heat transfer correlation of nanofluids, International Journal of Heat and Mass Transfer 43 (2000) 3701–3707.
[108] S.P. Jang, S.U.S. Choi, Free convection in a rectangular cavity (Benard convection) with nanofluids, in: Proceedings of IMECE, Anaheim, California, USA, 2004, pp. 13–19.
[109] Y. Xuan, Z. Yao, Lattice Boltzmann model for nanofluids, Heat and Mass Transfer/Waerme- und Stoffuebertragung 41 (3) (2005) 199–205.
[110] Y. Xuan, K. Yu, Q. Li, Investigation on flow and heat transfer of nanofluids by the thermal Lattice Boltzmann model, Progress in Computational Fluid Dynamics 5 (1/2) (2005) 13–19.